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Analysis of three-dimensional natural convection and entropy generation in a water filled open trapezoidal enclosure

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ABSTRACT

A three-dimensional numerical analysis of laminar natural convection with entropy generation in an open trapezoidal cavity filled with water has been carried out. In this investigation, the inclined wall is maintained at isothermal hot temperature while cold water enters into the cavity from its right open boundary and all other walls are assumed to be perfect thermal insulators. Attention is paid on the effects of buoyancy forces on the flow structure and temperature distribution inside the open enclosure. Rayleigh number is the main parameter which changes from 10^3 to 10^5 and Prandtl number is fixed at Pr =6.2. Obtained results have been presented in the form of particles trajectories, iso-surfaces of temperature and those of entropy generated as well as the average Nusselt number. It has been found that the flow structure is sensitive to the value of Rayleigh number and that heat transfer increases with increasing this parameter.

Keywords—*Natural convection; Entropy generation; Trapezoidal cavity; Rayleigh number; Nusselt number*

I. Introduction

The analysis of natural convection in both enclosed and open cavities has received significant attention due to many engineering applications, such as, electronic equipment cooling, heat loss from solar collectors, food processing and storage, building insulation, fire control, metallurgy and flow in nuclear reactors. In recent decades, there have been numerous studies on natural convection due to thermal buoyancy effects inside trapezoidal cavities. Iyican et al. [1,2] studied the natural convection motion and the heat transfer within a trapezoidal enclosure with parallel cylindrical top and bottom walls at different temperatures and plane adiabatic sidewalls. They presented both analytical and experimental results. Lam et al. [3] obtained similar results for a trapezoidal cavity having two vertical adiabatic walls, an inclined cold top wall and a hot floor. The effect of inclination angle of isothermal walls on flow structure for laminar natural convection flow in trapezoidal cavities was studied by Kuyper and Hoogendoorn [4]. They also analyzed the influence of Ra number on the average Nusselt number. Numerical investigation on natural convection within a partially divided trapezoidal cavity was carried out by Moukalled and Darwish [5]. Two different thermal boundary conditions were considered, case 1: the hot left wall and cold right wall and case 2: the hot right wall and cold left wall. It was found that the presence of the baffle decreases the overall heat transfer rate in the trapezoidal cavity, irrespective of the position and height of the baffle. They also observed that the maximum decrease in the heat transfer rate occurs for the baffle placed near to the left wall, irrespective of the baffle height. Moukalled and Darwish [6] further extended their study to investigate the effects of the height and position of the baffle protruding out from the inclined top wall, on the fluid flow and heat transfer in a trapezoidal enclosure with the similar thermal boundary conditions as mentioned in the earlier work [5]. Similarly, it was found that the overall heat transfer rate in the trapezoidal cavity is greatly reduced in the presence of the baffle, irrespective of the size and position of the baffle. The effect of the divider on natural convection in a partially divided trapezoidal enclosure under summer and winter boundary conditions was investigated by Arici and Sahin [7]. It was observed that there was no significant influence of the divider on the temperature distribution and consequently on the heat transfer rates during the summer boundary conditions. However, the heat transfer rate is reduced due to the presence of the divider during the winter boundary conditions. Silva et al. [8] analyzed the effect of the inclination angle of the top wall on the heat transfer rate in a trapezoidal cavity with two baffles placed on the cavity's horizontal surface. . It was found that, the temperature and velocity gradients within the enclosure decrease with the height of the baffles. They also concluded that for a fixed height of baffles, the overall heat transfer rate increases with the inclination angle. Two-dimensional trapezoidal cavities find big place in literature [9–19]. However, to the best of the authors' knowledge, no attention has been paid to investigate natural convection and entropy generation in partially or wholly open three-dimensional trapezoidal shaped spaces. Therefore, the main objective of the present study is to investigate numerically the laminar natural convection with entropy generation in an open three-dimensional trapezoidal cavity filled with water.

II. Mathematical Formulation

A. Physical model

Physical model is presented in fig. 1 with its specified coordinate system and boundary conditions. Indeed, the considered problem is three-dimensional natural convection and entropy generation in a one side opened trapezoidal cavity filled with water. The analyzed cavity is heated from inclined left wall and cooled from right open side while remaining walls are assumed to be insulated. The height, the length of the upper horizontal wall and the width of the cavity are equal to L while the length of the bottom wall is 2L.

B. Governing Equations and Numerical Solution

As numerical method we had recourse to the vorticity-potential vector formalism which allows, in a three-dimensional configuration, the elimination of the pressure, which is a delicate term to treat. To eliminate this term one applies the rotational to the equation of momentum. More details on this 3-D formalism can be found in the work of Kolsi et al. [20]. The potential vector and the vorticity are respectively defined by the two following relations:

$$\vec{\omega}' = \vec{\nabla} \times \vec{V}' \quad \text{and} \quad \vec{V}' = \vec{\nabla} \times \vec{\psi}' \tag{1}$$

$$-\vec{\omega} = \nabla^2 \vec{\psi} \tag{2}$$

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V}.\nabla)\vec{\omega} - (\vec{\omega}.\nabla)\vec{V} = \Delta\vec{\omega} + Ra.\mathrm{Pr}.\left[\frac{\partial T}{\partial z};0;-\frac{\partial T}{\partial x}\right] \tag{3}$$

$$\frac{\partial T}{\partial t} + \vec{V}.\nabla T = \Delta T \tag{4}$$

With: $\mathrm{Pr} = \frac{\nu}{\alpha}$ and $Ra = \frac{g.\beta.\Delta T.L^3}{\nu.\alpha}$

Boundary conditions for considered model are given as follows:

Temperature:

$T = 1$ on the left inclined wall and $T = 0$ on the right open side.

$\frac{\partial T}{\partial n} = 0$ on all other walls (adiabatic).

$T_{in} = 0$ if $n.V < 0$ at open boundary

$\left.\frac{\partial T}{\partial n}\right|_{out} = 0$ if $n.V \geq 0$ at open boundary

Velocity:

$V_x = V_y = V_z = 0$ on all walls

$\frac{\partial V_x}{\partial x} = \frac{\partial V_y}{\partial x} = \frac{\partial V_z}{\partial x} = 0$ at open boundary

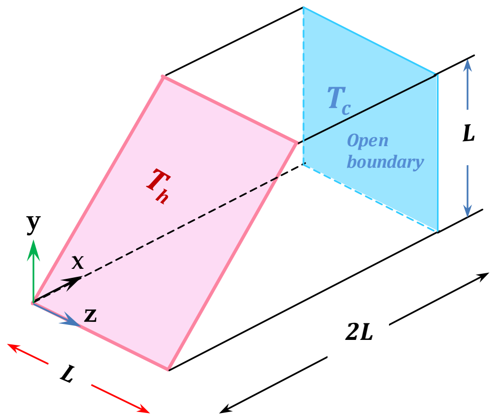


Fig. 1. Schematic of a water-filled open trapezoidal enclosure.

The generated entropy is written in the following form as:

$$S'_{gen} = -\frac{1}{T'^2}.\vec{q}.\vec{\nabla}T' + \frac{\mu}{T'}.\phi'$$

The first term represents the generated entropy due to temperature gradient and the second that due to the friction effects.

$$\vec{q} = -k.gra\vec{d}T$$

The dissipation function is written in incompressible flow as:

$$\phi' = 2\left[\left(\frac{\partial V'_x}{\partial x'}\right)^2 + \left(\frac{\partial V'_y}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial z'}\right)^2\right] + \left(\frac{\partial V'_y}{\partial x'} + \frac{\partial V'_x}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial y'} + \frac{\partial V'_y}{\partial z'}\right)^2 + \left(\frac{\partial V'_x}{\partial z'} + \frac{\partial V'_z}{\partial x'}\right)^2 \quad (5)$$

From where the generated entropy is written:

$$S'_{gen} = \frac{k}{T'^2_0}\left[\left(\frac{\partial T'}{\partial x'}\right)^2 + \left(\frac{\partial T'}{\partial y'}\right)^2 + \left(\frac{\partial T'}{\partial z'}\right)^2\right] + 2\frac{\mu}{T_0}\left\{\begin{array}{l}\left[\left(\frac{\partial V'_x}{\partial x'}\right)^2 + \left(\frac{\partial V'_y}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial z'}\right)^2\right] \\ + \left(\frac{\partial V'_y}{\partial x'} + \frac{\partial V'_x}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial y'} + \frac{\partial V'_y}{\partial z'}\right)^2 + \left(\frac{\partial V'_x}{\partial z'} + \frac{\partial V'_z}{\partial x'}\right)^2\end{array}\right\} \quad (6)$$

After adimensionalisation one obtains generated entropy number (dimensionless local entropy generated) which is written in the following way:

$$N_s = S'_{gen}\frac{1}{k}\left(\frac{LT_0}{\Delta T}\right)^2 \quad (7)$$

From where:

$$N_s = \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] + \varphi.\left\{2\left[\left(\frac{\partial V_x}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\right] + \left[\begin{array}{l}\left(\frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial y} + \frac{\partial V_y}{\partial z}\right)^2 \\ + \left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right)^2\end{array}\right]\right\} \quad (8)$$

With $\varphi = \frac{\mu\alpha^2 T_m}{L^2 k \Delta T^2}$ is the irreversibility coefficient.

The first term of N_s represents the local irreversibility due to the temperatures gradients, it is noted $N_{s\text{-}th}$. The second term represents the contribution of the viscous effects in the irreversibility it is noted $_{Ns\text{-}fr}$. N_s give a good idea on the profile and the distribution of the generated local dimensionless entropy. The total dimensionless generated entropy is written:

$$S_{tot} = \int_v N_s\, dv = \int_v \left(N_{s-th} + N_{s-fr}\right) dv = S_{th} + S_{fr} \quad (9)$$

The local and average Nusselt at the hot inclined wall are given by:

$$Nu = \frac{\partial T}{\partial n} \quad \text{and} \quad Num = \int_0^{\sqrt{2}L}\int_0^{L} Nu\, dn\, dz \quad (10)$$

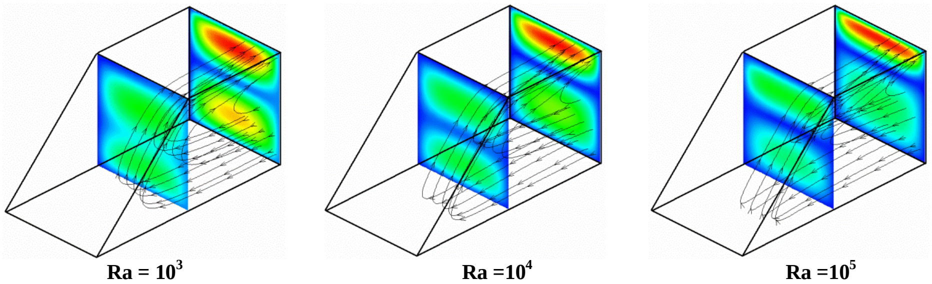
With: $\vec{n}$ is the unit vector normal to the hot inclined wall.

It should be noted that numerical analysis has been developed using an in-house computational code on the basis of FORTRAN programming language. The time step (10^{-4}) and spatial mesh ($122\times 61\times 61$) are utilized to carry out all the numerical tests. The solution is considered acceptable when the following convergence criterion is satisfied for each step of time:

$$\sum_i^{1,2,3}\frac{\max\left|\psi_i^n - \psi_i^{n-1}\right|}{\max\left|\psi_i^n\right|} + \max\left|T_i^n - T_i^{n-1}\right| \le 10^{-4} \quad (11)$$

III. Results And Discussion

Trajectories of particles and velocity magnitude for different Rayleigh number values are illustrated in fig. 2. It is noted that Prandtl number is fixed at Pr =6.2 for whole work and Rayleigh number is changed from 10^3 to 10^5. The numerical result shows an incoming water flow at the lower part of the right open side while the internal hotter water is pushed outwards through its upper part. Therefore, as seen from the figure, particles circulate in clockwise direction by absorbing heat from the heated side



Ra = 10^3 Ra = 10^4 Ra = 10^5

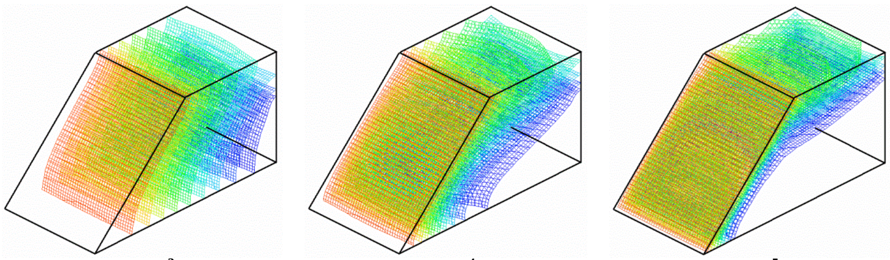
Fig. 2. Particles trajectory and velocity magnitude for different Rayleigh number values.

before leaving the cavity giving way to the cold particles to fill the cavity maintaining the cooling process. Thus a stream of water is produced with a front approaching the hot wall as the number of Rayleigh increases. Indeed, it can be noticed that the dead zone is reduced due to increasing of effectiveness of convection heat transfer. By increasing Ra, the heated and rising fluid leads to formation of the thermal boundary layer parallel to the inclined walls and there is a strong convective current resulting in a noticeable increase in the velocity magnitude at the upper part of the open side of the cavity while the left lower corner remains as stagnant zone. The highest magnitude of velocity occurred for Ra= 10^5.

Fig. 3 depicts the iso-surfaces of temperature for different Rayleigh number values. When the conduction is the dominant mode of heat transfer ($Ra = 10^3$), the isotherms present an almost vertical stratification and water is nearly at rest. It is obvious that these iso-surfaces are always orthogonal the adiabatic walls. By increasing Rayleigh number ($Ra \geq 10^4$), heated water near the hot wall is increasingly driven by the incoming water flow at the lower part of the cavity making inclined stratification near the heated side and horizontal stratification near the upper horizontal wall. Thus an excessive temperature gradient near the lower part of the hot wall took place. It should be noted that the entering water is cold and such penetration of low temperature wave leads to a formation of thin thermal boundary layer near the inclined hot wall. The latter characterizes a heating of the cavity upper part due to an appearance of ascending flow along the hot wall where cold water is heated. The mentioned thin thermal boundary layer illustrates the intensive motion of the water near the inclined wall.

It is noticed that the rate of heat transfer inside the enclosure is measured in term of the overall Nusselt number. Therefore, fig. 4 shows the variation of the average Nusselt number, which characterizes the heat transfer from the hot wall towards the vicinity of the enclosure, with the Rayleigh number. It is obvious that for low values of Ra and when the conduction is the dominant mode of heat transfer, this variation is insignificant. However, for heat removal from the heated inclined wall increases by means of increasing Rayleigh number and the maximum rate is obtained for the highest Ra as expected. Indeed, by increasing Rayleigh number to 10^5, the fluid flow intensifies and the thermal energy transport from the hot wall increases due to the enhancement of convection heat transfer.

To achieve a maximum heat transfer rate between the heated side and the cooling water, it is essential to carry out an entropy generation analysis to investigate the two sources of irreversibilities that are responsible for heat losses. These irreversibilities are mainly due to heat transfer and fluid friction.



Ra = 10^3 Ra = 10^4 Ra = 10^5

Fig. 3. Iso-surfaces of temperature for different Rayleigh number values.

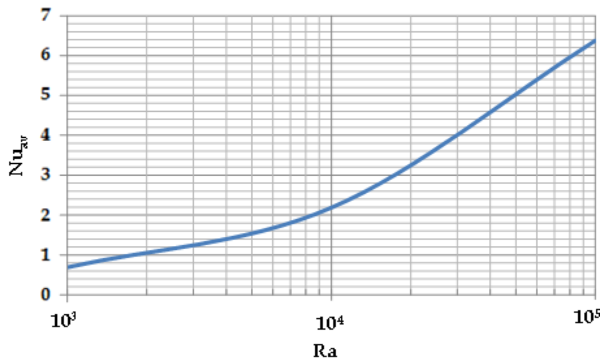


Fig. 4. Variation of Mean Nusselt number on hot wall with Rayleigh number.

Therefore, iso-surfaces of entropy generation (entropy generation due to heat transfer, entropy generation due to fluid friction and the total entropy generation) are shown in fig. 5 for an irreversibility coefficient and different values of Rayleigh number. It can be noticed that, for the lowest value of Rayleigh number ($Ra = 10^3$), the generated entropy covers the entire trapezoidal enclosure. However, for , the generated entropy concentrates (locates itself) along the inclined hot and upper horizontal walls. This result supports our observations made during the discussion on the particles trajectory and temperature distribution and can explain the boundary layer phenomenon met for the great Rayleigh number values. The maximum of entropy generation due to heat transfer S_{th} is located in the region near the center of the inclined hot wall. Moreover, it can be observed that the entropy generation due to heat transfer and total entropy generation follow nearly the same distribution which gives a clue that entropy due to heat transfer outweighs that due to fluid friction.

Such a result is supported by fig. 6 showing the entropy generation due to heat transfer, the entropy generation due to friction and the total entropy generation as function of Rayleigh number. In fact, irrespective the value of Ra, entropy generation due to viscous irreversibility is less significant in deciding the total entropy generation, which is sum of entropy generations due to heat transfer and friction.

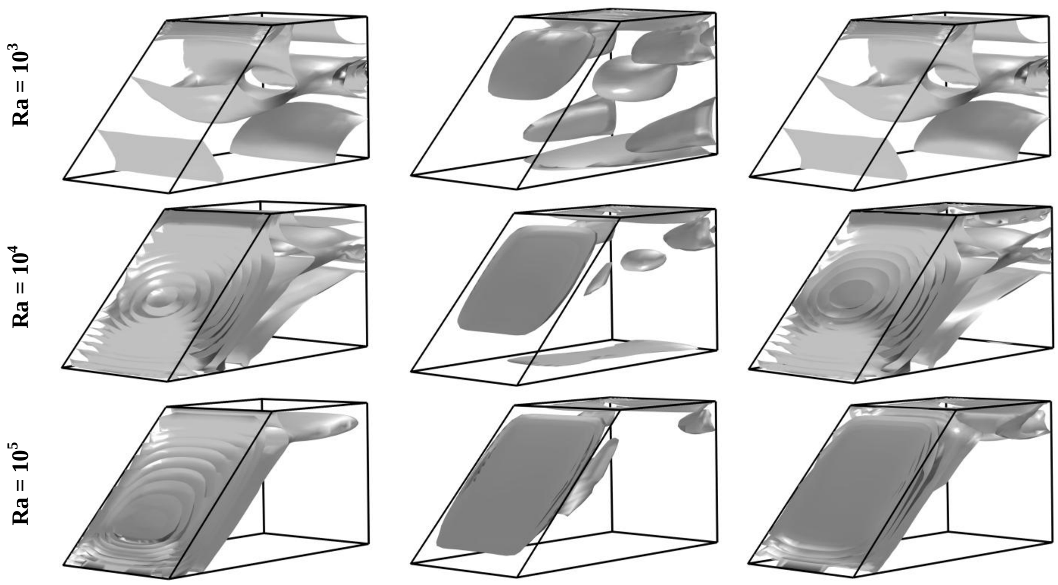


Fig. 5. Iso-surfaces of entropy generation a) Entropy generation due to heat transfer,
b) Entropy generation due to fluid friction, c) Total entropy generation for $\varphi = 10^{-5}$

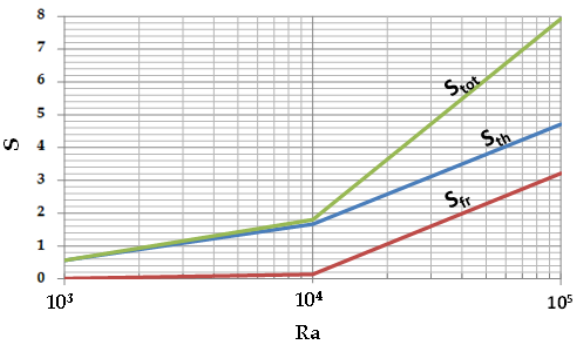


Fig. 6. Variation of entropy generation with Rayleigh number

IV. Conclusions

Three-dimensional numerical investigation has been carried out to simulate natural convection and entropy generation in a water filled open trapezoidal enclosure. Results are presented for different Rayleigh number values which is the main parameter of the study. Some conclusions can be drawn as follows:

- For lower values of Rayleigh number, conduction is the primary mode of heat transfer and the flow strength is very low due to poor convective heat transfer.
- Flow strength increases with increasing of Rayleigh number and a strong convective current is noticeable along the hot wall where cold water is heated.
- The increasing of Ra leads to a formation of thin thermal boundary layer near the inclined hot wall resulting in a heating of the cavity upper part due to an appearance of ascending flow along the hot wall where cold water is heated.
- The flow structure and the magnitude of velocity are sensitive to the value of Rayleigh number.
- Overall Nusselt number at the heated surface increases with increasing value of Ra indicating a maximum heat transfer rate at $Ra=10^5$.
- Irreversibilities are mainly due to heat transfer especially at low Rayleigh number values.

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