MHD Newtonian and non-Newtonian Nano Fluid Flow Passing On A Magnetic Sphere with Mixed Convection Effect

Basuki Widodo, Yolanda Norasia, Dieky Adzkiya
Department of Mathematics, Faculty of Mathematics, Computing, and Data Sciences
Institut Teknologi Sepuluh Nopember
Indonesia

ABSTRACT

This paper considers the problem of magneto-hydrodynamics (MHD) Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect. Nano Fluid is a combination of liquid fluid as a base fluid with small solid nano particles. Water is chosen as Newtonian base fluid and oil is chosen as non-Newtonian base fluid. Then, Alumina and Copper are chosen as solid particle in nano fluid. We further construct governing equation by applying continuity equation, momentum equation, and energy equation to obtain dimensional governing equations. The dimensional governing equations that have been obtained are converted into non-dimensional governing equations by substituting non-dimensional variables. The non-dimensional governing equations are further transformed into similarity equations using stream function and solved numerically using Euler Implicit Finite Difference method. We further analyse the effect of magnetic parameter towards velocity and temperature in MHD nano fluid flow. The results show that the increases of magnetic parameter impacts to the decrease of velocity and temperature. Then, the velocity and temperature of Newtonian nano fluid are higher than the velocity and temperature of non-Newtonian nano fluid. Also, the velocity and temperature of copper-water are higher than the velocity and temperature of Alumina-water.

Keywords*—Newtonian and non-Newtonian nano fluid; MHD; Sphere; Euler Implicit Finite Difference.*

I. INTRODUCTION

Nano fluid is a combination of liquid fluid as a base fluid with small solid nano particles [1]. Nano fluid is divided into two types, i.e. Newtonian nano fluid and non-Newtonian nano fluid. Newtonian nano fluid is a base fluid in nano fluid which has a linear relationship between viscosity and shear stress. However, non-Newtonian nano fluid is the opposite of Newtonian nano fluid. In this paper, water is chosen as Newtonian base fluid and oil is chosen as non-Newtonian base fluid. Then, Alumina (Al_2O_3) and Copper (Cu) are chosen as solid particle in nano fluid. Alumina (Al_2O_3) contains metal oxide and copper (Cu) contains metal. These types of fluids are used in industrial area that needs for heating and cooling based on heat transfer [2].

Because of those, we conduct a research how to analyse MHD nano fluid flow problem using numerical simulation based on mathematical modelling. Putra et al [3] have illustrated the natural convection of nano-fluids. Their investigations stated that the thermal conductivity of solid nano particles can be increased when mixed with base fluid. Wen and Ding [4] have discussed about experimental investigation into convection heat transfer of nano fluids at the entrance region under laminar flow conditions. Akbar et al [5] have investigated unsteady MHD nano fluid flow through a channel with moving porous walls and medium by using Runge Kutta. The results show that the heat transfer rate increases and mass transfer rate decreases with the increase of Reynolds number. Mahat et al [6] also have observed mixed convection boundary layer flow past a horizontal circular cylinder in visco-elastic nano fluid with constant wall temperature and solved numerically by using the Keller-Box method. The results indicate that the velocity and temperature are increased by increasing the values of nano particles volume fraction and mixed convection parameter. Juliyanto et al [7] also have solved the problem of the effect of heat generation on mixed convection in nano fluids over a horizontal circular cylinder numerically by using Keller-Box method. The result of their investigations show that the velocity increase and temperature decrease when mixed convection parameter increases. In the present paper, we are interested to develop mathematical modelling of the problem of MHD newtonian and non-newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect. The influence of magnetic parameter (M), mixed convection parameter (λ), and volume fraction (χ) towards velocity and temperature in Newtonian and non-Newtonian nano fluid are investigated.

II. MATHEMATICAL FORMULATION

The unsteady MHD Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect is considered. *Fig. 1*illustrates the physical model of the problem and the coordinate system used to develop the mathematical model. The fluid used is Newtonian nano fluid and non-Newtonian nano fluid. The bluff body used is a magnetic sphere with radius *a*. The flow of nano fluid is assumed laminar flow and incompressible. The magnetic Reynolds number is assumed to be very small. Therefore, there is no electrical voltage which makes electric field. With potential theory, where the velocity potential is perpendicular with stream function, so the 3D dimensional governing equations can be transform into 2D dimensional governing equations.

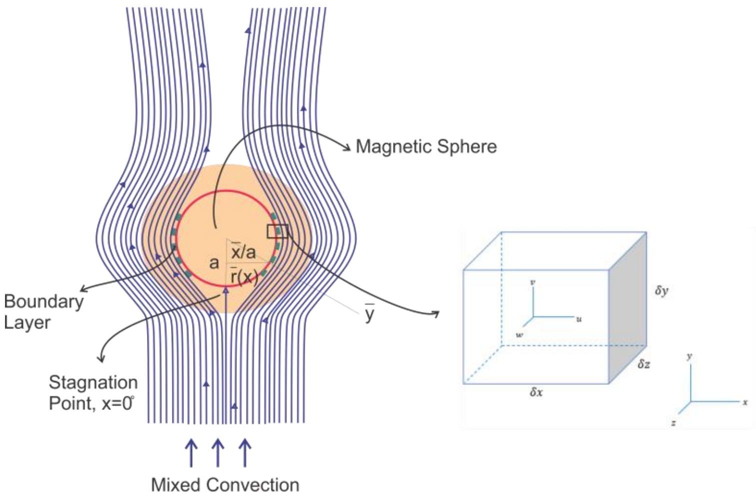

Fig. 1 Physical model and coordinate system

Based on the physical model and coordinate system, unsteady MHD Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere is illustrated in *Fig. 1*. The 2D dimensional governing equations are developed from the law of conservation mass, the second law of Newton, and the first law of Thermodynamics. We further obtain continuity equation, momentum equation, and energy equation, which can be written as follows:

Continuity Equation:

$$\frac{\partial \bar{r}\bar{u}}{\partial \bar{x}} + \frac{\partial \bar{r}\bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

Momentum Equation :
at x axis

$$\rho_{fn}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu_{fn}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \sigma B_0^2 \bar{u} - \rho_{fn}\beta(\bar{T} - T_\infty)g_{\bar{x}} \quad (2)$$

at y axis

$$\rho_{fn}\left(\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)=-\frac{\partial\bar{p}}{\partial\bar{x}}+\mu_{fn}\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)+\sigma B_0^2\bar{u}-\rho_{fn}\beta(\bar{T}-T_\infty)g_{\bar{y}} \quad (3)$$

Energy Equation :

$$\left(\frac{\partial\bar{T}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right)=\alpha_{fn}\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \quad (4)$$

With the initial and boundary condition as follows :

$\bar{t}=0: \bar{u}=\bar{v}=0, \bar{T}=T_\infty$, for every $\bar{x},\bar{y}$
$\bar{t}>0: \bar{u}=\bar{v}=0, \bar{T}=T_w$, for $\bar{y}=0$
$\bar{u}=\bar{u}_e(\bar{x}), \bar{u}=\bar{v}=0, \bar{T}=T_\infty$ as $\bar{y}\to\infty$

where ρ_{fn} is density of nano fluid, μ_{fn} is dynamic viscosity of nano fluid, g is the gravitational acceleration, and α_{fn} is thermal diffusivity of nano fluid. In addition, the value of r is defined as $\bar{r}(\bar{x})=a\sin(\bar{x}/a)$.

Further, the 2D dimensional governing equations (1)-(4) are transformed into non-dimensional equations by using both non-dimensional parameters and variables. In this problem, the non-dimensional variables are given as in [7], i.e.:

$$x=\frac{\bar{x}}{a}; y=Re^{1/2}\frac{\bar{y}}{a}; t=\frac{U_\infty\bar{t}}{a}; u=\frac{\bar{u}}{U_\infty}$$

$$v=Re^{1/2}\frac{\bar{u}}{U_\infty}; r(x)=\frac{\overline{r(x)}}{a}$$

where $g_{\bar{x}}$ and $g_{\bar{y}}$ are defined as in [7]

$$g_{\bar{x}}=-g\,sin\left(\frac{\bar{x}}{a}\right)$$
$$g_{\bar{y}}=g\,cos\left(\frac{\bar{x}}{a}\right)$$

Boundary layer theory [8] is applied to non-dimensional governing equation. We obtain the following results

Continuity Equation

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \quad (5)$$

Momentum Equation

at x axis

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{v_{nf}}{v_f}\frac{\partial^2 u}{\partial y^2}+Mu+\lambda T\sin x \quad (6)$$

at y axis

$$-\frac{\partial p}{\partial y}=0 \quad (7)$$

Energy Equation

$$\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{1}{Pr}\frac{\alpha_{fn}}{\alpha_f}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

where these nano fluid constants are defined as [9], i.e. :

Density of nanofluid :

$$\rho_{fn} = (1-\chi)\rho_f + \chi\rho_s$$

Dynamic viscosity :

$$\mu_{nf} = \mu_f \frac{1}{(1-\chi)^{2.5}}$$

Specific heat :

$$(\rho C_p)_{nf} = (1-\chi)(\rho C_p)_f + \chi(\rho C_p)_s$$

Heat conductivity :

$$k_{nf} = \frac{k_s + 2k_f - 2\chi(k_f - k_s)}{k_s + 2k_f + \chi(k_f - k_s)} k_f$$

The thermo-physical properties of nano particles and base fluid is given in Table 1 [10].

TABLE I. THERMO-PHYSICAL PROPERTIES

Properties	***Water***	***Oil***	***Cu***	Al_2O_3
density	997.1	884	8933	3970
specific heat of constant pressure	4179	1900	385	765
thermal conductivity	0.613	0.145	400	40

We substitute those nano fluid constants into (6) and (8). We obtain

Momentum Equation :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{1}{(1-\chi)^{2.5}} \frac{1}{(1-\chi)+\chi\left(\frac{\rho_s}{\rho_f}\right)}\right)\frac{\partial^2 u}{\partial y^2} + Mu + \lambda T \sin x \quad (9)$$

And

Energy equation :

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{Pr}\frac{k_s+2k_f-2\chi(k_s-k_f)}{k_s+2k_f+\chi(k_s-k_f)} \frac{1}{(1-\chi)+\chi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\frac{\partial^2 T}{\partial y^2} \quad (10)$$

Further, by converting (9) and (10) into non-similarity equations using stream function, which is given as follows [11]

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}$$

$$v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$$

Where

$$\psi = t^{\frac{1}{2}} u_e(x) r(x) f(x,\eta,t),$$

$$\eta = {}^{y}\big/_{t^{\frac{1}{2}}},$$

$$T = s(x,\eta,t)$$

The equation (9) and (10) are modified by substituting stream function as follows:

Momentum Equation :

$$\left[\frac{1}{(1-\chi)^{2.5}\left[(1-\chi)+\left(\frac{\rho_s}{\rho_f}\right)\right]}\right]\frac{\partial^3 f}{\partial\eta^3}+\frac{\eta\,\partial^2 f}{2\partial\eta^2}+t\frac{\partial u_e}{\partial x}\left[1-\left(\frac{\partial f}{\partial\eta}\right)^2+f\frac{\partial^2 f}{\partial\eta^2}\right)\right]=t\frac{\partial^2 f}{\partial\eta\partial t}+tu_e\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial\eta^2}-\frac{1}{r}\frac{\partial r}{\partial x}f\frac{\partial^2 f}{\partial\eta^2}\right)+ Mt\left(1-\frac{\partial f}{\partial\eta}\right)-\frac{\lambda st}{u_e}\sin x \quad (11)$$

Energy Equation :

$$\left(\frac{k_s+2k_f-2\chi(k_s-k_f)}{k_s+2k_f+\chi(k_s-k_f)}\frac{1}{(1-\chi)+\chi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\right)\frac{\partial^2 s}{\partial\eta^2}+Pr\frac{\eta}{2}\frac{\partial s}{\partial\eta}+\Pr t\frac{\partial u_e}{\partial x}f\frac{\partial s}{\partial\eta}=Pr\,t\left[\frac{\partial s}{\partial\eta}+u_e\left(\frac{\partial f}{\partial\eta}\frac{\partial s}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial s}{\partial\eta}-\frac{1}{r}\frac{\partial r}{\partial x}f\frac{\partial s}{\partial\eta}\right)\right]$$

(12)With the initial and boundary condition are as follows :

$t = 0 :\ f = \frac{\partial f}{\partial\eta} = s = 0$untuksetiap$x,\eta$

$t > 0 :\ f = \frac{\partial f}{\partial\eta} = 0\,,s = 1$ketika$\eta = 0$

$\frac{\partial f}{\partial y} = 1\,,s = 0$ketika$\eta \to \infty$

By substituting local free stream for sphere case [12],$u_e = \frac{3}{2}\sin x$ into (11) and (12) respectively, we obtain

Momentum Equation :

$$\left[\frac{1}{(1-\chi)^{2.5}\left[(1-\chi)+\left(\frac{\rho_s}{\rho_f}\right)\right]}\right]\frac{\partial^3 f}{\partial\eta^3}+\frac{\eta\,\partial^2 f}{2\partial\eta^2}+\frac{3}{2}t\cos x\left[1-\left(\frac{\partial f}{\partial\eta}\right)^2+2f\frac{\partial^2 f}{\partial\eta^2}\right)\right]=t\frac{\partial^2 f}{\partial\eta\partial t}+\frac{3}{2}\,t\sin x\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial\eta^2}\right)+ Mt\left(1-\frac{\partial f}{\partial\eta}\right)-\frac{2}{3}\lambda st \quad (13)$$

Energy Equation :

$$\left(\frac{k_s+2k_f-2\chi(k_s-k_f)}{k_s+2k_f+\chi(k_s-k_f)}\frac{1}{(1-\chi)+\chi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\right)\frac{\partial^2 s}{\partial\eta^2}+Pr\frac{\eta}{2}\frac{\partial s}{\partial\eta}+3\cos x Pr\,t\,f\frac{\partial s}{\partial\eta}=Pr\,t\frac{\partial s}{\partial\eta}+\Pr t\frac{3}{2}\sin x\left(\frac{\partial f}{\partial\eta}\frac{\partial s}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial s}{\partial\eta}\right) \quad (14)$$

III. Numerical Procedures

MHD Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect have been investigated numerically by using Euler Implicit Finite Difference method. he set of similarity equation and boundary condition are discretized by a second order central difference method and solved by a computer program which has been developed.

Momentum Equation :

$$\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]\frac{\partial^2 u}{\partial\eta^2}+\frac{\eta}{2}\frac{\partial u}{\partial\eta}+\frac{3}{2}\,t\left(1-(u)^2+f\,\frac{\partial u}{\partial\eta}\right)=t\,\frac{\partial u}{\partial t}+Mt\,(1-u\,)-\frac{2}{3}\,\lambda st$$

by using Euler implicit finite difference method we obtain

$$\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]\frac{1}{\Delta\eta^2}(u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1})+\frac{\eta_i}{2}\left(\frac{3u_{i+1}^{n+1}-4u_i^{n+1}+u_{i-1}^{n+1}}{2\Delta\eta}\right)$$
$$+\frac{3}{2}t^{n+1}\left(1-(u_i^{n+1})^2+2\frac{1}{2\Delta\eta}f_i^n(3u_{i+1}^{n+1}-4u_i^{n+1}+u_{i-1}^{n+1})\right)$$
$$=t^{n+1}\frac{1}{2\Delta t}(3u_i^{n+1}-4u_i^n+u_i^{n-1})+Mt^{n+1}(1-u_i^{n+1})-\frac{2}{3}\lambda s_i^n t^{n+1}$$

where K_i

$$K_i=\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]\frac{1}{\Delta\eta^2}(u_{i+1}^n-2u_i^n+u_{i-1}^n)+\frac{\eta_i}{4}\frac{1}{\Delta\eta}(3u_{i+1}^n-4u_i^n+u_{i-1}^n)$$
$$+\frac{3}{2}t^{n+1}\left(1-(u_i^n)^2+\frac{f_i^n}{\Delta\eta}(3u_{i+1}^n-4u_i^n+u_{i-1}^n)\right)-t^{n+1}M\,(1-u_i^n)+\frac{2}{3}\lambda s_i^n t^{n+1}$$

and for

$$A_0=\frac{1}{4}\frac{\eta_i}{\Delta\eta}+\frac{3}{2}t^{n+1}\frac{f_i^n}{\Delta\eta}$$

$$A_1=\frac{\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]}{\Delta\eta^2}+A_0$$

$$A_2=\frac{3}{2}\frac{t^{n+1}}{\Delta t}+2\frac{\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]}{\Delta\eta^2}-t^{n+1}M+3\,t^{n+1}u_i^n+4A_0$$

$$A_3=\frac{\left[\frac{1}{(1-\chi)^{2.5}((1-\chi)+\chi\frac{\rho_s}{\rho_f})}\right]}{\Delta\eta^2}+3A_0$$

Energy Equation :

$$\Pr t\,\frac{\partial s}{\partial t}=\left[\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))((1-\chi)+\left(\frac{\chi\,\rho\,(Cp)_s}{\rho\,(Cp)_f}\right)}\right]\frac{\partial^2 s}{\partial\eta^2}+Pr\frac{\eta}{2}\frac{\partial s}{\partial\eta}+3\Pr t\;f\;\frac{\partial s}{\partial\eta}$$

by using implicit finite difference method we get

$$\Pr t^{n+1}\frac{1}{2\Delta t}(3s_i^{n+1}-4s_i^n+s_i^{n-1})$$
$$=\left[\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))((1-\chi)+\left(\frac{\chi\,\rho\,(Cp)_s}{\rho\,(Cp)_f}\right)}\right]\frac{1}{\Delta\eta^2}(s_{i+1}^{n+1}-2s_i^{n+1}+s_{i-1}^{n+1})$$
$$+Pr\frac{\eta_i}{\Delta\eta}\frac{1}{2}(3s_{i+1}^{n+1}-4s_i^{n+1}+s_{i-1}^{n+1})+3\Pr t^{n+1}f_i^n\frac{1}{2\Delta\eta}(3s_{i+1}^{n+1}-4s_i^{n+1}+s_{i-1}^{n+1})$$

Where L_i

$$L_i = \left[\frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{((k_s + 2k_f) + \chi(k_f - k_s))((1-\chi) + \left(\frac{\chi\,\rho(Cp)_s}{\rho(Cp)_f}\right)}\right]\frac{1}{\Delta\eta^2}(s_{i+1}^n - 2s_i^n + s_{i-1}^n)$$
$$+\frac{1}{4}Pr\frac{\eta_i}{\Delta\eta}(3s_{i+1}^n - 4s_i^n + s_{i-1}^n) + 3\,\mathrm{Pr}\frac{t^{n+1}}{2\Delta\eta}f_i^n(3s_{i+1}^n - 4s_i^n + s_{i-1}^n)$$

and for

$$B_0 = \frac{1}{4}\frac{\eta_i}{\Delta\eta} + \frac{3}{2}Pr\,t^{n+1}\frac{f_i^n}{\Delta\eta}$$

$$B_1 = \frac{\left[\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))((1-\chi)+\left(\frac{\chi\,\rho(Cp)_s}{\rho(Cp)_f}\right)}\right]}{\Delta\eta^2} + B_0$$

$$B_2 = \frac{3}{2}Pr\frac{t^{n+1}}{\Delta t} + 2\frac{\left[\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))((1-\chi)+\left(\frac{\chi\,\rho(Cp)_s}{\rho(Cp)_f}\right)}\right]}{\Delta\eta^2} + 4B_0$$

$$B_3 = \frac{\left[\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))((1-\chi)+\left(\frac{\chi\,\rho(Cp)_s}{\rho(Cp)_f}\right)}\right]}{\Delta\eta^2} + 3B_0$$

IV. Results and Discussion

In this research, the effect of magnetic parameter (M) to velocity and temperature in Newtonian and non-Newtonian nano fluid are analyzed.

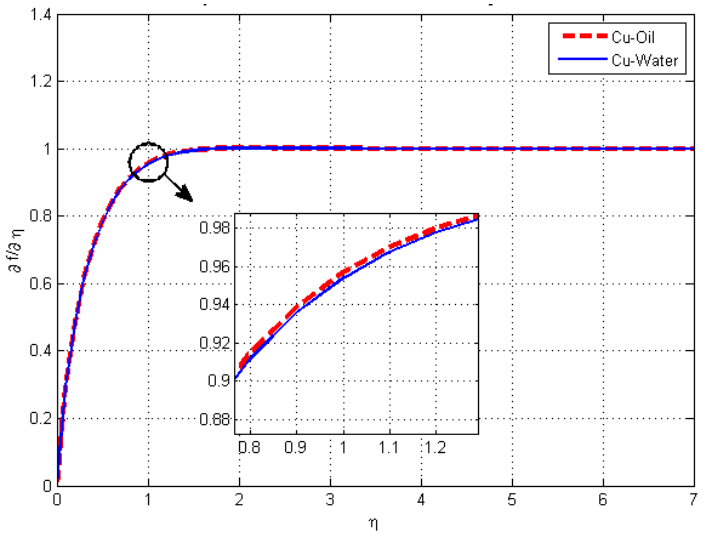

Fig. 2 Velocity Profile of Cu-Oil and Cu-Water with Magnetic Influence

Water is chosen as Newtonian base fluid and oil is chosen as non-Newtonian base fluid. Then, Alumina (Al_2O_3) and Copper (*Cu*) are chosen as solid particle in nano fluid. The numerical results of the velocity and temperature with respect to the position in front of the lower stagnation at the point $x = 0^{\circ}$with value of magnetic parameters $M = 1$ are depicted in *Fig. 2* and *Fig. 3* respectively.

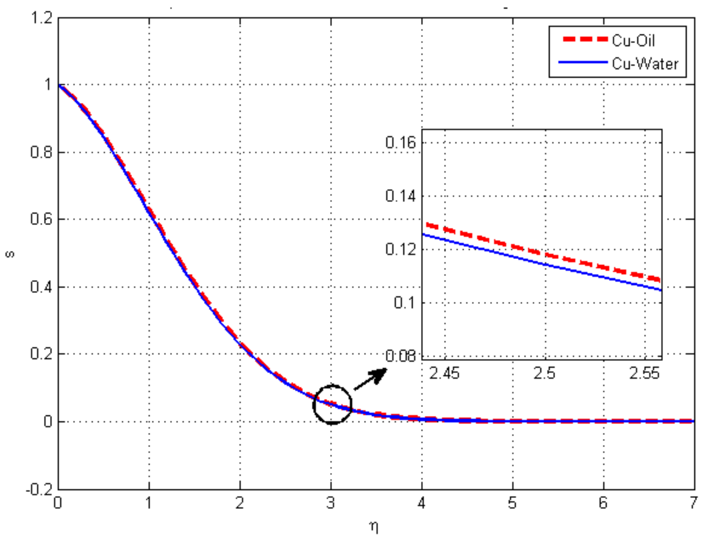

Fig. 3 Temperature Profile of Cu-Oil and Cu-Water with Magnetic Influence

Fig. 2 shows the velocity profiles of the MHD Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect. *Cu*-oil is used as non-Newtonian nano fluid and *Cu*-water is used as Newtonian nao fluid. The results show that the velocity of Newtonian nano fluid is higher than the velocity of non-Newtonian nano fluid. Also, *Fig. 3* shows that the temperature of Newtonian nano fluid is higher than the temperature of non-Newtonian nano fluid.

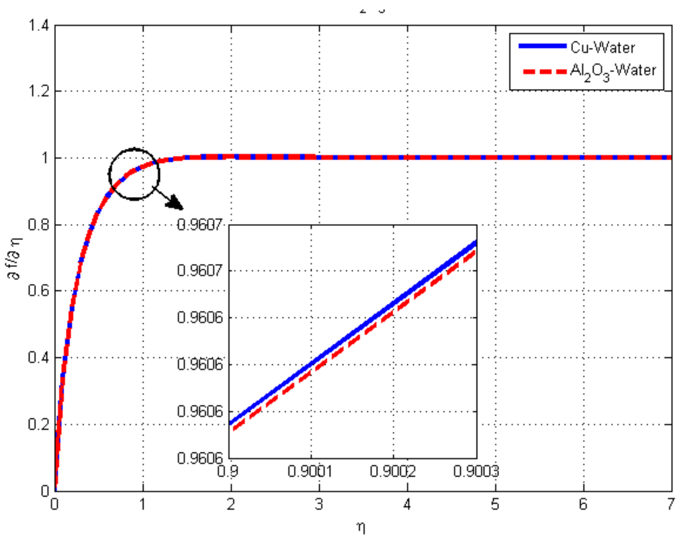

Fig. 4 Velocity Profile of Cu-Water and Al_2O_3-Water with Magnetic Influence

Fig. 4 and *Fig. 5* show the velocity profiles and temperature profiles of *Cu*-Water and Al_2O_3-water respectively. Alumina (Al_2O_3) contains metal oxide and copper (*Cu*) contains metal. The results show that the velocity of *Cu*-Water is higher than the velocity of Al_2O_3-water. *Fig. 5* also shows that the temperature of *Cu*-water is higher than the temperature of Al_2O_3-water.

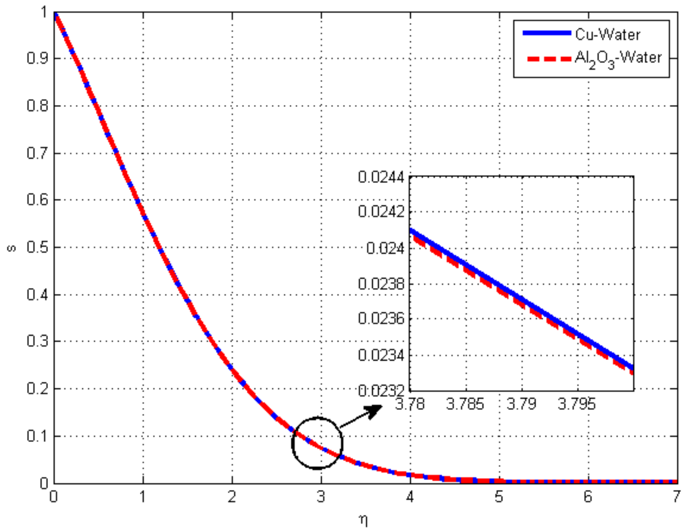

Fig. 5 Temperature Profile of Cu-Water and Al_2O_3-Water with Magnetic Influence

The numerical results of the velocity and temperature in Newtonian nano fluid *Cu*-Water with respect to the position in front of the lower stagnation at the point $x = 0^{\circ}$ with various value of magnetic parameters $M = 0, 1, 3,$ and 5are illustrated in *Fig. 6* and *Fig. 7* respectively.

Fig. 6 shows the velocity profiles of the MHD Newtonian nano fluid *Cu*-Water flow passing on a magnetic sphere at various M when mixed convection parameter $\lambda = 1$and volume fraction $\chi = 0.1$. The results show that velocity and temperature of Newtonian nano fluid *Cu*-Water in *Fig. 6* and *Fig. 7* decrease when magnetic parameter increases. The magnetic parameter represents the presence of Lorentz force in a magnetic sphere. Therefore, when magnetic parameter increases, then the Lorentz force also increases. It impacts to decrease of the velocity and temperature in Newtonian nano fluid *Cu*-Water.

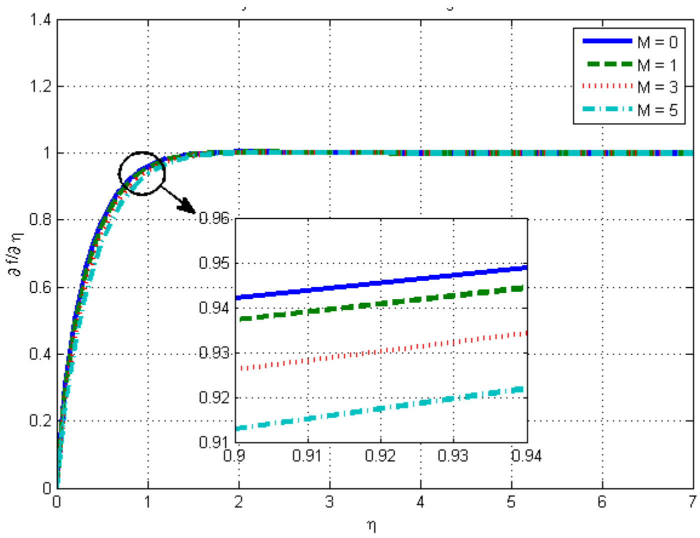

Fig. 6 Velocity Profile for various *M of* Cu-Water

The numerical results of the velocity and temperature in non-Newtonian nano fluid *Cu*-Oil with respect to the position in front of the lower stagnation at the point $\chi = 0.1$with various value of magnetic parameters $M = 0, 1, 3,$ and 5 are illustrated in *Fig. 8* and *Fig. 9* respectively.

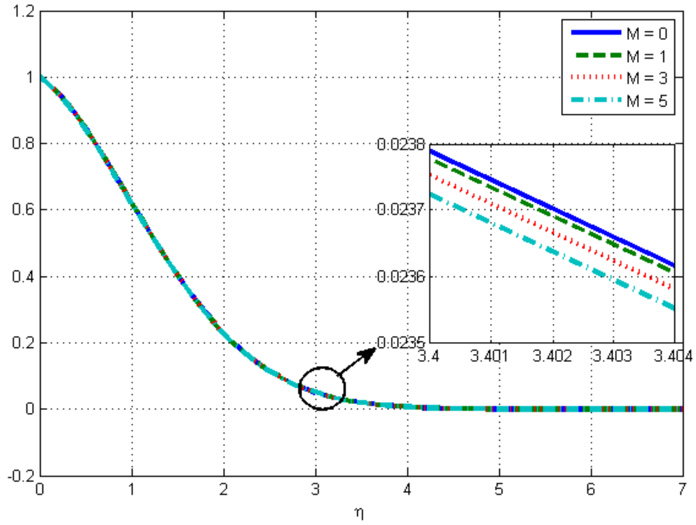

Fig. 7 Temperature Profile for various *M of* Cu-Water

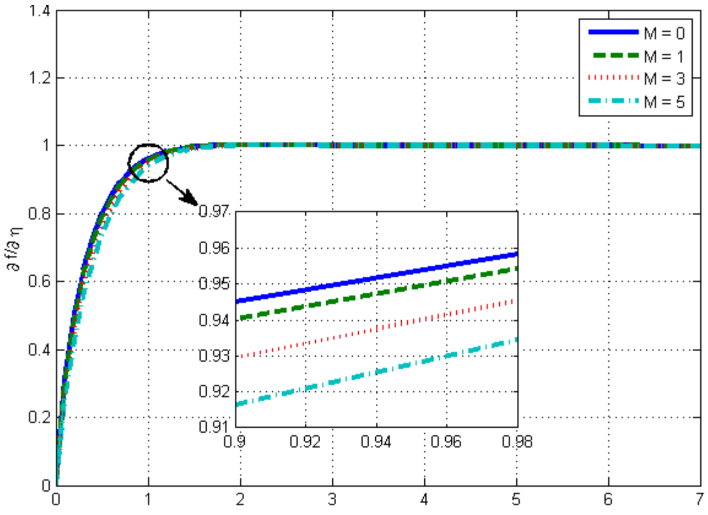

Fig. 8 Velocity Profile for various *M of* Cu-O

Fig. 8 shows the velocity of the MHD non Newtonian nano fluid *Cu*-Oil flow passing on a magnetic sphere at various M when mixed convection parameter $\lambda = 1$ and volume fraction $\chi = 0.1$. *Fig. 9* shows the temperature of the MHD non Newtonian nano fluid *Cu*-Oil flow passing on a magnetic sphere at various M when mixed convection parameter $\lambda = 1$and volume fraction $\chi = 0.1$.The results in *Fig. 8* and *Fig. 9* show that velocity profiles and temperature profiles of non-Newtonian nano fluid *Cu*-Oil decrease when magnetic parameter increases.

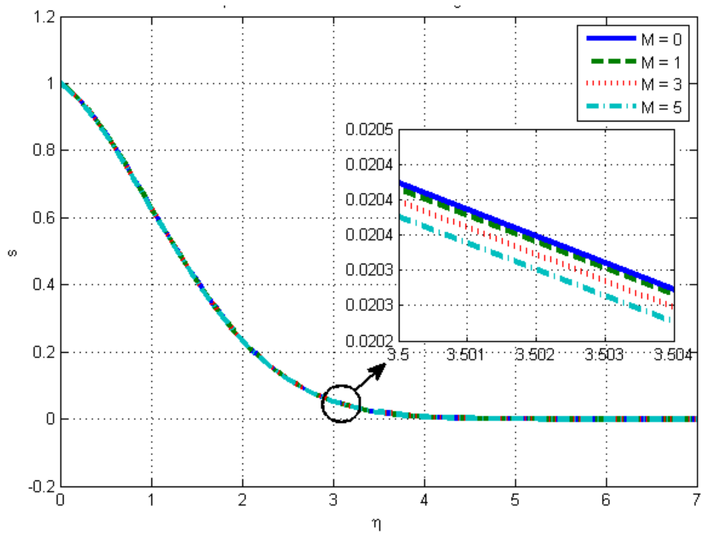

Fig. 9 Temperature Profile for various *M of* Cu-Oil

V. Conclusions

MHD Newtonian and non-Newtonian nano fluid flow passing on a magnetic sphere with mixed convection effect have been investigated numerically by using Euler Implicit Finite Difference method. We have considered water as Newtonian base fluid and oil is chosen as non-Newtonian base fluid. Further, Alumina (Al_2O_3) and Copper (*Cu*) are chosen as solid particle in nano fluid. We further obtain numerical results that when effects of magnetic parameter, mixed convection parameter, and volume fraction are included, the velocity and temperature profiles change. It is concluded that the velocity and temperature of Newtonian nano fluid *Cu*-water are higher than the velocity and temperature of non-Newtonian nanofluid *Cu*-Oil. The velocity and temperature of copper-water *Cu*-Water also are higher than the velocity and temperature of Alumina-water Al_2O_3-Water. Further, the velocity profiles and temperature profiles of Newtonian *Cu*-Water and non-Newtonian nano fluid *Cu*-Oil decrease when magnetic parameter increases.

Acknowledgment

This research is supported by the Institute for Research and Community Services, InstitutTeknologiSepuluhNopember (ITS) Surabaya, Indonesia with Funding Agreement Letter number 970/PKS/ITS/2018. We further are very grateful to LPPM-ITS for giving us a chance to submit this paper in an International Journal.

References

[1] Y. Ding, Y. Chen, Y. He, A. Lapkin, M.Yeganeh, L.Siller, Y. V.Butenko, "Heat Transfer Intensification Using Nanofluids", Advanced Powder Technol, 2007, pp.813-24.

[2] S. Das, S. Chakraborty, R. N. Jana, O. D. Makinde, "Entropy analysis of unsteady magneto-nanofluid flow past accelerating stretching sheet with convective boundary condition" Appl. Math. Mech., 2015, pp.1593-610.

[3] Putra N, Roetzel W, Das S K , "Natural convection of nano-fluids", Heat and Mass Transfer,2003, pp.775-84.

[4] D. Wen and Y. Ding, "Experimental Investigation into Convective Heat Transfer of Nanofluids at the Entrance Region under Laminar Flow Conditions",International journal of heat and mass transfer, 2004, pp.5181-8.

[5] M. Z. Akbar, M. Ashraf, M. F. Iqbal, K. Ali,"Heat and mass transfer analysis of unsteady MHD nanofluid flow through a channel with moving porous wall and medium.", AIP Conf Proc, 2016, 045222.

[6] R. Mahat, N. A. Rawi, A. R. M. Kasim, S. Shafie, "Mixed convection boundary layer flow past a horizontal circular cylinder in viscoelastic nanofluid with constant wall temperature",Malaysian Journal of Fundamental and Applied Sciences, 2017, 13(4-1).

[7] B. Juliyanto, B. Widodo, C. Imron, "The effect of heat generation on mixed convection flow in nano fluids over a horizontal circular cylinder",IOP Conf. Series, 2018, 012001

[8] B. Widodo,I. Anggriani, C. Imron, "The Characterization of Boundary Layer Flow in The Magnetohydrodynamic Micropolar Fluid Past A Solid Sphere", ,International Journal of Advances in Science Engineering and Technology, 2016, pp. 2321-9009.

[9] A. Karimipour, A. H. Nezhad, A. D'Orazio, E. Shirani, "The effects og inclinatipn angle amd Prandtl number on the mixed convection in the inclined lid driven cavity using lattice Boltzmann method", Journal of Theoretical and Applied Mechanics, 2013, pp.447-62.

[10] A. Zaib, A. R. M. Kasim, N. F. Mohammad, S. Shafie, "Unsteady MHD Mixed Convection Stagnantion Point Flow in A Micropolar Fluid on A Vertical Surface in A Porous Medium with Soret and Dufour Effects",Heat Transfer Research, 2013, pp.603-20.

[11] B. Widodo, D. K. Arif, D. Aryany, N. Asiyah, F. A. Widjajati,Kamiran, "The effect of magnetohydrodynamic nano fluid flow through porous cylinder", AIP Conference Proceedings, 2017, 020069.

[12] N.Freidoonimehr, M. M. Rashidi, S. Mahmud, "Unsteady MHD free convective flow past a permeable stretching vertical surface in a nano-fluid", International Journal of Thermal Sciences, 2015, pp.136-45.