Study of Chemical ion Transport through the Soil using combined Variational Iteration and Homotopy Perturbation Methods.

Liberty Ebiwareme, Iyai Davies
Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.

Fun-Akpo Pere Kormane
Department of Civil Engineering, Rivers State University, Port Harcourt, Nigeria.

ABSTRACT

The aim of this present article is to solve numerically the resulting governing equation in the modelling of the chemical ion transport through the soil. The governing equation was transformed to a nonlinear ordinary differential equation through similarity transformation. The hybrid semi-analytical coupling of the variational iteration and Homotopy perturbation method was proposed to solve the resulting equation. The effect of the pertinent parameters of the flow namely, the porosity parameter, Hartmann number and Reynold number on the concentration, velocity profiles and volumetric flow rate are analysed, and the result presented in tables and graphs. The result obtained revealed, this hybrid method is computationally easy, convenient, and showed promise to solve most highly nonlinear differential equation. The parameters significantly have impacts on the system.

Keywords:Chemical ion, Variational Iteration method (VIM), Homotopy Perturbation method (HPM), Variational Homotopy Perturbation Method (VHPM)

I. INTRODUCTION

Most of the real-life phenomena of physical significance in science, engineering, social sciences, biology, agriculture, and others are modelled in the form of differential or partial differential equations which are inherently nonlinear in nature. Due to this nonlinearity, solutions of equations governing these phenomena are difficult to obtain. Academics over the years have proposed several methods to finding for their solutions ranging from numerical and analytical. The nature of the complexity posed by the nonlinearityenable academics to choose numerical methods as the best method for solving them. Examples of some of the earliest numerical methods employed include: Runge-Kutta, Crank-Nicolsen, Euler's, Modified Euler, Finite difference method, Finite element method and Finite volume method [1-2].

In contemporary times, due to the advent of powerful computers with incredible capabilities and symbolic computational software'sin mathematics like Mathematica, Maple, MATLAB and COMSOLE Multiphysics, these problems can be solved at a canter regardless of the degree of nonlinearity. After numerical methods lost its appeal in tackling these problems, next comes weighted residual methods (Galerkin, Petrov-Galerkin, Moment method, Collocation method, least square method, and subdomain method) and variational method (Rayleigh-Ritz) have been used to obtain solutions to these equations. Depending on the nature of the nonlinearity, most of the methodsmentioned above are difficult to implement and not computationally convenient and time consuming Babolian et al [3].

Most recently, semi-analytical methods which give the solution in a series of convergent iterative step is fast gaining popularity among academics due to their ease of implementation and computational convenience. Solutions using these methods agree with exact solutions with high degree of accuracy. Examples of these methods include: Adomian decomposition method (ADM), Differential transformation method (DTM), Homotopy perturbation method (HPM), Variational iteration method (VIM), Differential Quadrature method (DQM), Akbari-Ganji method (AGM), Homotopy Analysis method (HAM), Exp-Function method. Others are the combination of two or more of these methods, notable among these ones includes: Laplace Variation Iteration method (LVIM), Laplace Adomian decomposition method (LADM), Optimal Homotopy Asymptotic method (OHAM), Spectral Homotopy Analysis method (SHAM), Spectral Homotopy perturbation method (SHPM), Optimal Homotopy perturbation method (OHPM), Laplace Homotopy Perturbation method (LHPM) and Variational Homotopy perturbation method (VHPM) [4-10].

The variational Homotopy perturbation method (VHPM) is a semi-analytical method which combines the variational iteration and Homotopy perturbation methods This method requires firstly by taking the variational iteration method of the given system under consideration to give an iterative system. Secondly, applying the Homotopy perturbation to the resulting system and equating corresponding powers of the perturbation parameter, give different set of equations of the unknowns. Solving this equation give the solution in approximate form. This method though nascent has been extensively applied in solving several problems in both science and engineering. Malinfar and Mahdavi [11] have examined the application of the variational Homotopy perturbation method on the generalized Fisher's equation. Allahviranloo et al [12] have investigated variational Homotopy perturbation method as an efficient iterative scheme in solving partial differential equations in fluid mechanics. The approximate solution of the foam drainage equation with time and space fractional derivative have been studied by Bouhassoun et al. [13]. Ji-Huan [28] used the coupled Variational iteration and Homotopy perturbation techniques for solving nonlinear problems. Ebiwareme and Kormane [14] have employed VHPM to solve analytically the nonlinear equations governing MHD Jeffery-Hamel flow in the presence of magnetic field. The study showed that that the velocity profile is significantly affected for both the convergent/divergent channel for different values of the Reynold number, magnetic field, and angle of inclination. Fredholm integrodifferential equations of fractional order have been analysed using combined variational iteration and Homotopy perturbation methods [15]. Linear and nonlinear heat transfer equations in engineering have been investigated using VHPM [16]. Yangqin [17] used VHPM to solve fractional initial boundary value problems. Mohyud-in and Mohammed [18-19] have studied higher order dimensional initial boundary value problems utilizing VHPM

The motivation in this present study is to propose the novelty of applying variational Homotopy perturbation method to solve the nonlinearequations governing the transport of chemical ion through the soil. Theorganization of the study is as follows: Section 2 presents the formulation of the problem with the accompanying governing equations and boundary conditions. The fundamentals of variation iteration and Homotopy perturbation methods are given in section 3 and 4. Basics of variational Homotopy perturbation method which is the coupling of VIM and HPM is presented in section 5. Section 6 give the detailed mathematical analysis of the problem using VHPM. In the final analysis, the results and discussion of the obtained solution is graphically presented in section 7 while the conclusion of the study is contained in section 8 with the major findings in the study itemized.

II. MATHEMATICAL FORMULATION

We consider a steady Newtonian fluid flow in a porous channelandsandwiched between two parallel plates under the influence of transverse external pressure applied at both ends. Then centre of the channel parallel to the channel surface is represented by the $x-a$xis, whereas the transverse direction is along the $y-$axis. The flow is assumed to be asymmetrical about the $x-$axis and the channel walls are represented by $y = h$ and $y = \delta h$, where h denotes the width of the channel.

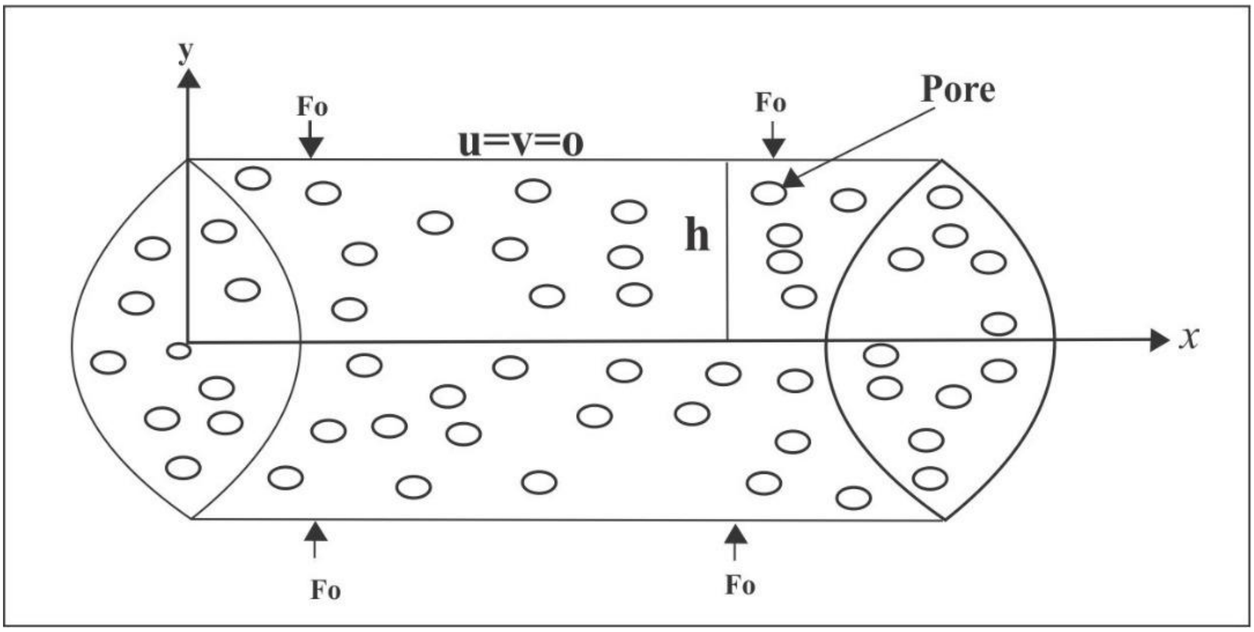

Figure 1. Schematic configuration of the problem (Adopted from Poonam et al. 2019)

Let (u, v) be the velocity component of the fluid along the x and y axes and F_0 be the external force applied to the top and bottom plates. Assuming the pressure gradient to be zero, then the governing equationsof continuityand momentum for the boundary layer of incompressible fluid is given by

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{\kappa}u - \frac{F_0^2 u}{\rho} \quad (2)$$

Where ν, ρ, κ denote the kinematic coefficient of viscosity, density of the solute and permeability of the porous medium respectively

Since the flow of the fluid is symmetric along the central line, $y = 0$ of the channel. We therefore focus our attention to the flow region, $0 \leq y \leq h$, where the suitable boundary conditions are given by

$$u = v = 0 \text{ at } y = h \quad (3)$$

$$\frac{\partial u}{\partial y} = 0, v = 0 \quad \text{at } y = 0 \quad (4)$$

Using the Non-dimensional parameters of the form

$$\varepsilon = \frac{x}{h}, \eta = \frac{y}{h}, u = \frac{U_0 x f'(\eta)}{h}, v = -U_0 f(\eta) \quad (5)$$

Where U_0 is the characteristic velocity

Using Eq. (5) into Eqs. (1) and (2), we obtain the governing equations in non-dimensional form as

$$f''(\eta) + Re\left(f(\eta)f'(\eta) - \left(f'(\eta)\right)^2\right) - \frac{\nu}{\kappa}f'(\eta) = 0 \quad (6)$$

Where Re and κ are the Reynold and porosity permeability parameter defined by

$$Re = \frac{U_0 h}{\nu}, \mathcal{K} = \frac{\kappa}{h^2}, \psi = (F_0^2 + K^{-1}) \quad (7)$$

Subject to the boundary conditions

$$f(1) = f'(1) = 0$$
$$f(0) = f''(0) = 0 \quad (8)$$
$$f'(0) = \alpha$$

III. VARIATIONAL ITERATION METHOD (VIM)

Consider the ordinary differential equation of the form

$$Ly + N(y) = f(x), \qquad x \in I \quad (9)$$

Where L and N are linear and nonlinear operators respectively, and $f(x)$ is any given inhomogeneous terms defined for $x \in I$

We defined a correctional functional for Eq. (8) as follows

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda(\xi)\left(Ly_n(\xi) + N(\tilde{y}_n(\xi)) - f(\xi)\right)d\xi \quad (10)$$

Where $\lambda(\tau)$ is a Lagrange multiplier obtained through variational theory, $y_n(x)$ is the nth approximation of $y(x)$ and $\tilde{y}_n(x)$ is a restricted variation meaning $\delta\tilde{y}_n(x) = 0$

By imposing the variation of both sides of Eq. (9) and taking the restricted variation we obtained

$$\delta y_{n+1}(x) = \delta y_n(x) + \delta(\int_0^x \lambda(\xi) Ly_n(\xi)d\xi \quad (11)$$

$$\delta y_{n+1}(x) = \delta y_n(x) + [\lambda(\xi)(\int_0^\tau Ly_n(\xi)d\xi]_{\xi=0}^{\xi=x} - \int_0^x \lambda'(\xi)(\int_0^\tau L\delta y_n(\xi))d\xi \quad (12)$$

Now by applying the stationary condition, the value of the Lagrange multiplier, $\lambda(\xi)$ can be found. Then the successive approximations, $y_n(x), n = 0,1,2,3$ .... Can be found in the form

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda(\xi)\left(Ly_n(\xi) + N(y_n(\xi)) - f(\xi)\right)d\,\xi \quad (13)$$

The exact solution is then obtained as the limit of the successive approximations from Eq. (12)

$$y(x) = \lim_{n\to\infty} y_n(x) \quad (14)$$

IV. HOMOTOPY PERTURBATION METHOD (HPM)

In this section, the fundamentals of the Homotopy perturbation method as proposed by He. J. Huan is discussed

Consider a functional differential equation of the form

$$\mathcal{A}(u) - f(r) = 0, r \in \Omega \quad (15)$$

Subject to the boundary condition

$$\mathcal{B}\left(u, \frac{\partial u}{\partial t}\right) = 0, r \in \mathcal{T} \quad (16)$$

Where$\mathcal{A}$ is a differential operator, $\mathcal{B}$ is a boundary operator, $\mathcal{B}$ is a boundary operator, $\mathcal{T}$ is the boundary of the domain $\Omega, f(x,t)$ is a known analytic function and $u(x,t)$ is an unknown function

Dividing the operator, $\mathcal{A}$ into two parts comprising linear, $(\mathcal{L})$ and nonlinear $(\mathcal{N})$

$$\mathcal{A} = \mathcal{L} + \mathcal{N} \quad (17)$$

In view of Eq. (16), we rewrite Eq. (15) in the form

$\mathcal{L}(u) + \mathcal{N}(u) - f(r) = 0$ (18)

Embedding an artificial parameter p on Eq. 18) as follows

$\mathcal{L}(u) + p(\mathcal{N}(u) - f(r)) = 0$ (19)

where $p \in [0,1]$ is the embedding or artificial parameter.

Next, we construct a Homotopy, $\mathcal{H}(r,p): \Omega \times [0,1] \longrightarrow \Re$ to Eq. (19) that satisfies

$\mathcal{H}(r,p) = (1-p)[\mathcal{L}(v) - \mathcal{L}(u_0)] + p[\mathcal{L}(v) + \mathcal{N}(v) - f(r)] = 0$ (20)

and

$\mathcal{H}(r,p) = \mathcal{L}(v) - \mathcal{L}(u_0) + p\mathcal{L}(u_0) + p[\mathcal{N}(v) - f(r)] = 0$ (21)

Where $u_0(x)$ is the initial approximation which satisfies the boundary condition.

Putting $p = 0$ and $p = 1$ into Eq. (20), we obtain the following equations

$$\left.\begin{array}{l}\mathcal{H}(r,0) = \mathcal{L}(v) - \mathcal{L}(u_0)\\ \mathcal{H}(r,1) = \mathcal{A}(u) - f(r)\end{array}\right\} \quad (22)$$

Clearly as p changes monotonically from zero to unity, $\mathcal{H}(r,p)$ changes from $u_0(x)$ to $u(x)$. This is called deformation, whereas the terms $\mathcal{L}(v) - \mathcal{L}(u_0)$ and $\mathcal{A}(u) - f(r)$ are homotopic to each other.

Now we consider a power series solution in p as follows

$v = \sum_{n=0}^{\infty} p^{(n)} v_n$ (23)

The approximate solution of Eq. (23) can be obtained by setting $p = 1$

$u(x) = \lim_{p\to 1} v_n = v_0 + v_1 + v_2 + \cdots$

(24)

Similarly, the nonlinear term, $\mathcal{N}(u)$ can be expressed as He's polynomial

$\mathcal{N}(u) = \sum_{n=0}^{\infty} p^{(m)} H_m(v_0 + v_1 + \cdots + v_m)$ (25)

Where $H_m(v_0 + v_1 + \cdots + v_m) = \frac{1}{m!}\frac{\partial^m}{\partial p^m}[\mathcal{N}(\sum_{k=0}^{m} p^k v_k)]_{p=0}, m = 0,1,2,\ldots$ (26)

where,

$$H_0 = \mathcal{N}(u_0)$$

$$H_1 = u_1\mathcal{N}'(u_0)$$

$$H_2 = u_2\mathcal{N}'(u_0) + \frac{1}{2}\mathcal{N}_1^2\mathcal{N}''(u_0)$$

$$H_3 = u_3\mathcal{N}'(u_0) + u_1u_2\mathcal{N}''(u_0) + \frac{1}{6}\mathcal{N}_1^3\mathcal{N}'''(u_0)$$

$$H_4 = u_4\mathcal{N}'(u_0) + (\tfrac{1}{2}u_2^2 + u_1u_3)\mathcal{N}''(u_0) + \tfrac{1}{2}u_1^2u_2\mathcal{N}_1^3\mathcal{N}'''(u_0) + \tfrac{1}{24}u_4^3\mathcal{N}^{(iv)}(u_0) \quad (27)$$

V. VARIATIONAL HOMOTOPY PERTURBATION METHOD (VHPM)

To implement the VHPM, firstly we construct the correctional functional for Eq. (6) is given as

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^{\eta} \lambda(\xi)\left[\frac{d^3 f_n(\xi)}{d\eta^3} - \psi\frac{df_n(\xi)}{d\eta} + Re\left(f_n(\xi)\frac{df_n(\xi)}{d\eta} - \left(\frac{df_n(\xi)}{d\eta}\right)^2\right)\right]d\xi, n \geq 0 \quad (28)$$

where $\lambda(\xi)$ is the Lagrange multiplier which is obtained via optimal variation. Next, we apply the Homotopy perturbation to the functional, we get.

$$\sum_{n=0}^{\infty} p^{(n)}u_n = f_0(\eta) + p\int_0^{\eta}\frac{(\tau-\eta)^2}{2}\left[\frac{d^3u}{d\eta^3} - \psi\frac{du}{d\eta} + Re(\sum_{n=0}^{\infty}p^{(n)}A_n) - (\sum_{n=0}^{\infty}p^{(n)}B_n)\right]$$

(29)

Taking $u = \sum_{n=0}^{\infty} p^{(n)} u_n = u_0 + pu_1 + p^2u_2 + \cdots$. (30)

Substitution into the above expression, we obtain

$$\sum_{n=0}^{\infty} p^{(n)}u_n = f_0(\eta) + p\int_0^{\eta}\frac{(\tau-\eta)^2}{2}\left[\left(\frac{d^3u_0}{d\eta^3} + p\frac{d^3u_1}{d\eta^3} + p^2\frac{d^3u_2}{d\eta^3} + \cdots\right) - \psi\frac{d}{d\eta}\left(\frac{du_0}{d\eta} + p\frac{du_1}{d\eta} + p^2\frac{du_2}{d\eta} + \cdots\right) + Re(\sum_{n=0}^{\infty}p^{(n)}A_n) - (\sum_{n=0}^{\infty}p^{(n)}B_n)\right] \quad (31)$$

VI. ANALYTICAL PROCEDURE VIA VHPM

In this section, we analyse eq. (15) subject to the boundary conditions in Eq. (16) using the variational Homotopy perturbation method. This proposed method is the fusion of two semi-analytical methods viz, variational iteration method (VIM) and Homotopy perturbation method (HPM).

The correction functional for Eq. (6) is given by

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^\eta \lambda(\xi)\left[\frac{d^3 f_n(\xi)}{d\eta^3} - \psi\frac{df_n(\xi)}{d\eta} + Re\left(f_n(\xi)\frac{df_n(\xi)}{d\eta} - \left(\frac{df_n(\xi)}{d\eta}\right)^2\right)\right]d\xi \quad (32)$$

Now, we apply Homotopy Perturbation method to the correction functional in Eq. (9), we have

$$\sum_{n=0}^{\infty} p^{(n)} u_n = f_0(\eta) + p\int_0^\eta (\xi-\eta)\left[\frac{d^3}{d\eta^3}(u_0 + pu_1 + p^2u_2 + \cdots) - \psi\frac{d}{d\eta}(u_0 + pu_1 + p^2u_2 + \cdots) + Re\left(\sum_{n=0}^{\infty} p^{(n)} A_n\right) - \left(\sum_{n=0}^{\infty} p^{(n)} B_n\right)\right] \quad (33)$$

Where A_n and B_n are called the He's polynomial defined as follows

$$A_n = \frac{1}{k!}\frac{\partial^k}{\partial\lambda^k}\left[N\left(\sum_{n=0}^{\infty} f_n\lambda^n\right)\right]_{\lambda=0},\ k = 0,1,2,3\ldots \quad (34)$$

$$B_n = \frac{1}{k!}\frac{\partial^k}{\partial\lambda^k}\left[N\left(\sum_{n=0}^{\infty} f_n\lambda^n\right)\right]_{\lambda=0}, k = 0,1,2,3\ldots \quad (35)$$

The first six terms of the He's polynomials are explicitly defined as follows

$$\begin{aligned}
A_0 &= f_0 f_0' \\
A_1 &= f_0 f_1' + f_1 f_0' \\
A_2 &= f_0 f_2' + f_1 f_1' + f_2 f_0' \\
A_3 &= f_0 f_3' + f_1 f_2' + f_2 f_1' + f_3 f_0' \\
A_4 &= f_0 f_4' + f_1 f_3' + f_2 f_2' + f_3 f_1' + f_4 f_0' \\
A_5 &= f_0 f_5' + f_1 f_4' + f_2 f_3' + f_3 f_2' + f_4 f_1' + f_5 f_0' \\
A_6 &= f_0 f_6' + f_1 f_5' + f_2 f_4' + f_3 f_3' + f_4 f_2' + f_5 f_1' + f_6 f_0'
\end{aligned} \quad (36)$$

$$\begin{aligned}
B_0 &= \left(f_0'\right)^2 \\
B_1 &= 2f_0' f_1' \\
B_2 &= 2f_0' f_2' + \left(f_1'\right)^2 \\
B_3 &= 2f_0' f_3' + 2f_1' f_2' \\
B_4 &= \left(f_2'\right)^2 + 2f_1' f_3' + 2f_0' f_4' \\
B_5 &= 2f_0' f_5' + 2f_1' f_4' + 2f_2' f_3' \\
B_6 &= \left(f_3'\right)^2 + 2f_0' f_6' + 2f_1' f_5' + 2f_2' f_4'
\end{aligned} \quad (37)$$

Equating the coefficients of the like powers of p, we obtain the resulting equations as follows

$$p^{(0)}:\ f_0(\eta) = f(0) + \eta f'(0) + \frac{\eta^2}{2} f''(0) \quad (38)$$

$$p^{(1)}:\ f_1(\eta) = \int_0^\eta \frac{(\tau-\eta)^2}{2}\left[Re(A_0 - B_0) - \psi f_0'\right] d\tau \quad (39)$$

$$p^{(2)}:\ f_2(\eta) = \int_0^\eta \frac{(\tau-\eta)^2}{2}\left[Re(A_1 - B_1) - \psi f_1'\right] d\tau \quad (40)$$

Plugging Eqs. (35)– (36) into Eq. (37)–(39), we obtain the first three iterative solutions as

$$f_0(\eta) = \alpha\eta \quad (41)$$

$$f_1(\eta) = \frac{1}{6}Re\alpha^2\eta^4 - \frac{\alpha}{6}(Re+\psi)\eta^3 \quad (42)$$

$$f_2(\eta) = -\frac{\alpha}{6}[2Re^2\alpha(1+\phi) + (Re+\phi)]\eta^5 + \frac{2}{9}Re\alpha^2[Re(\alpha-1)+1]\eta^6 + \frac{5}{36}Re^2\alpha^3\eta^7 \quad (43)$$

Using the limiting relation, we compute the three-term approximation for the solution as

$$f(\eta) = \lim_{n\to\infty} f_n(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots \quad (44)$$

$$f(\eta) = \alpha\eta - \frac{\alpha}{6}(Re+\psi)\eta^3 + \frac{1}{6}Re\alpha^2\eta^4 - \frac{\alpha}{6}[2Re^2\alpha(1+\phi) + (Re+\phi)]\eta^5 + \frac{2}{9}Re\alpha^2[Re(\alpha-1)+1]\eta^6 + \frac{5}{36}Re^2\alpha^3\eta^7 \quad (45)$$

Setting $Re = 50, \psi = 5$ and using the condition, $f(1) = 0$, we obtain the value of the constant, $\alpha = 0.9905$

Substituting the value of $\alpha = 0.9905$ back into Eq. (44) yields the approximate solution given by

$$f(\eta) = 0.9905\eta - 0.165083(Re+\psi)\eta^3 + 0.163515Re\eta^4 + \cdots \quad (46)$$

DifferentiatingEq. (45) for the velocity profile and using the condition, $f'(1) = 0$, we obtain the constant as $\alpha = 0.995$

The volumetric flow rate, V_m is given by

$$V_m = 2\int_0^1 f'(\eta)d\eta$$

$$V_m = 2\int_0^1 (0.995 - 0.4975(Re+\psi)\eta^2 + 0.660017Re\eta^3)d\eta \quad (47)$$

VII. RESULTS AND DISCUSSION

In this section, we present the result of the problem for the temperature as well the velocity profile graphically for variations in porosity and Reynold numbers. The result show both porosity and Reynold numbers significantly impacted the flow. Similarly for the volumetric flow rate, an increase or decrease leads to a corresponding increase or decrease.

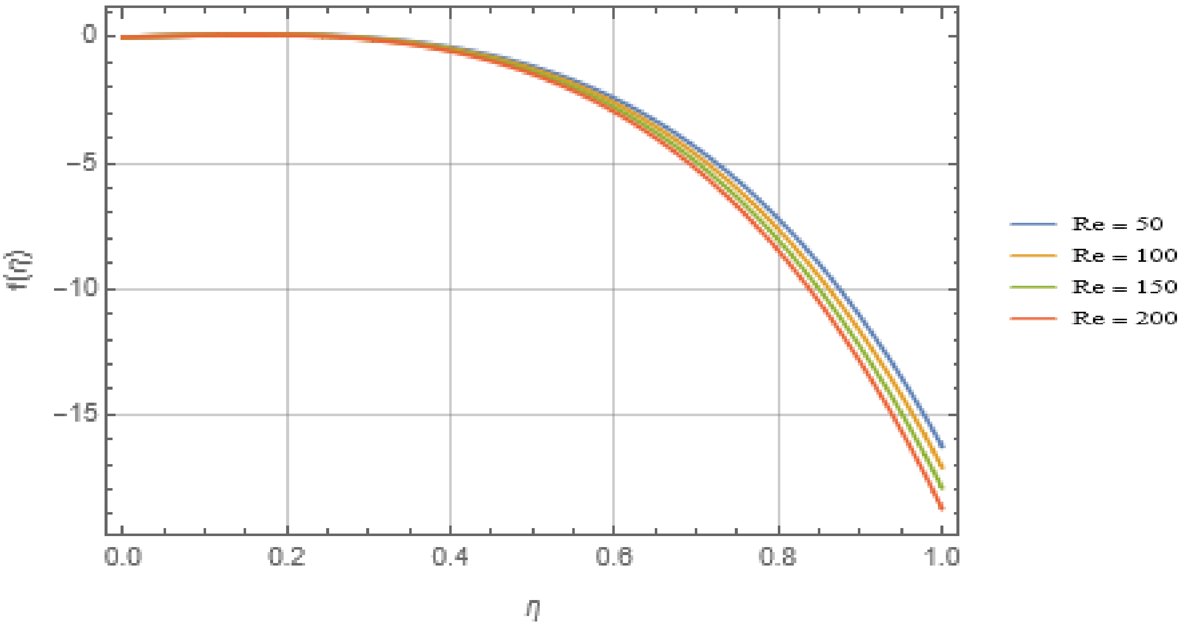

Figure 1. Variation of $f(\eta)$ with η for different values of Reynold number

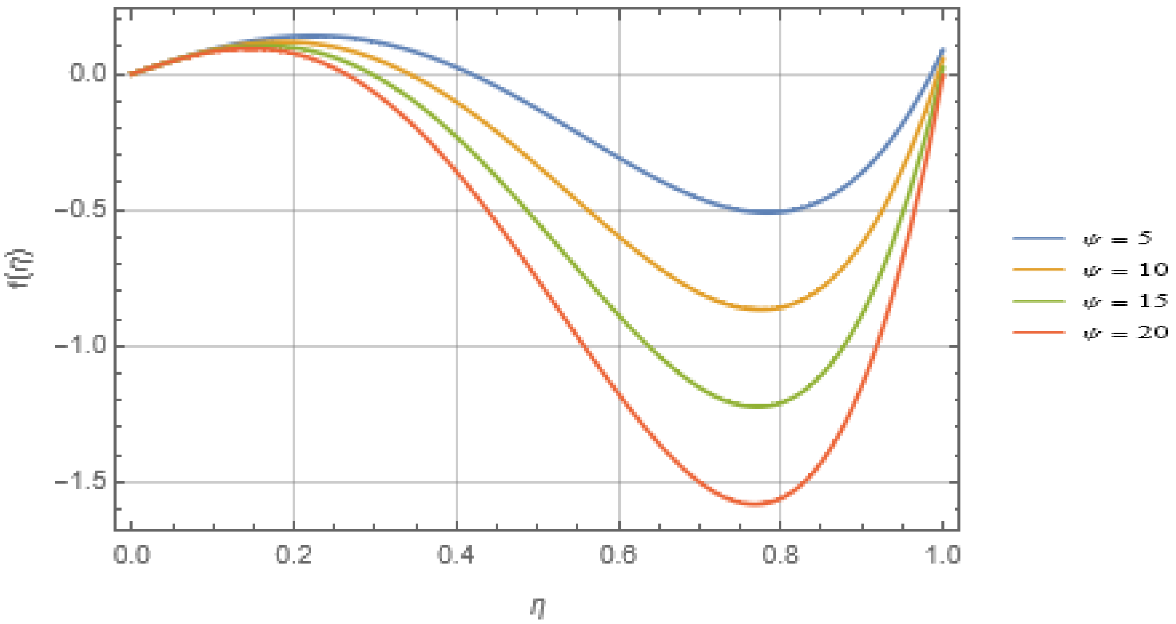

Figure 2. Variation of $f(\eta)$ with η for different values of porosity parameter

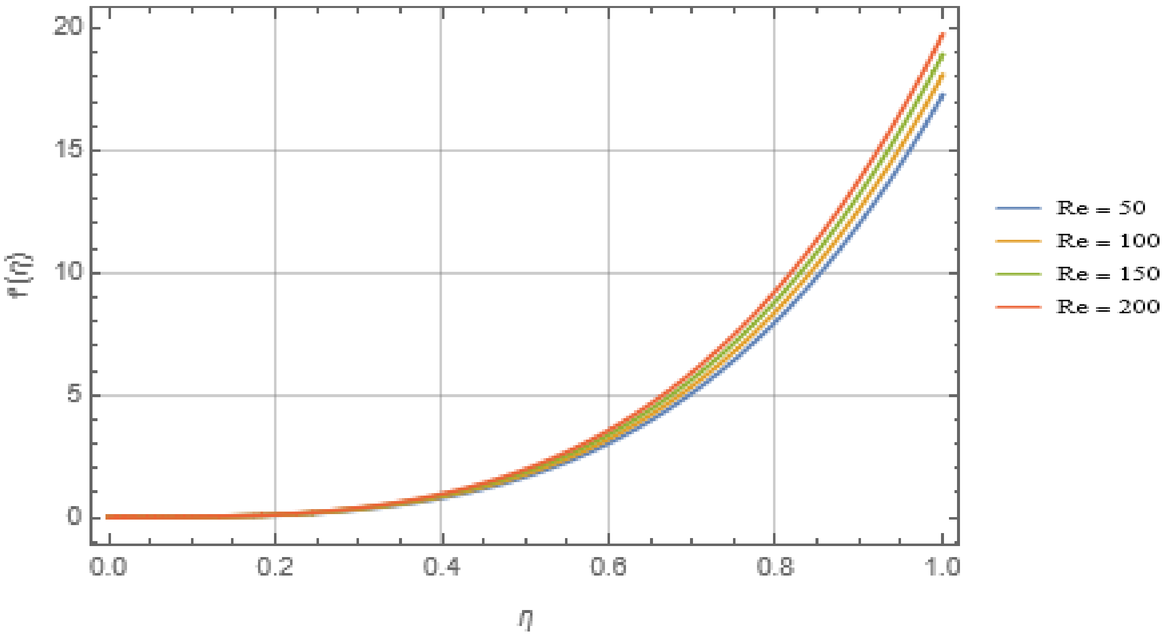

Figure 3. Variation of $f'(\eta)$ with η for different values of Reynold number

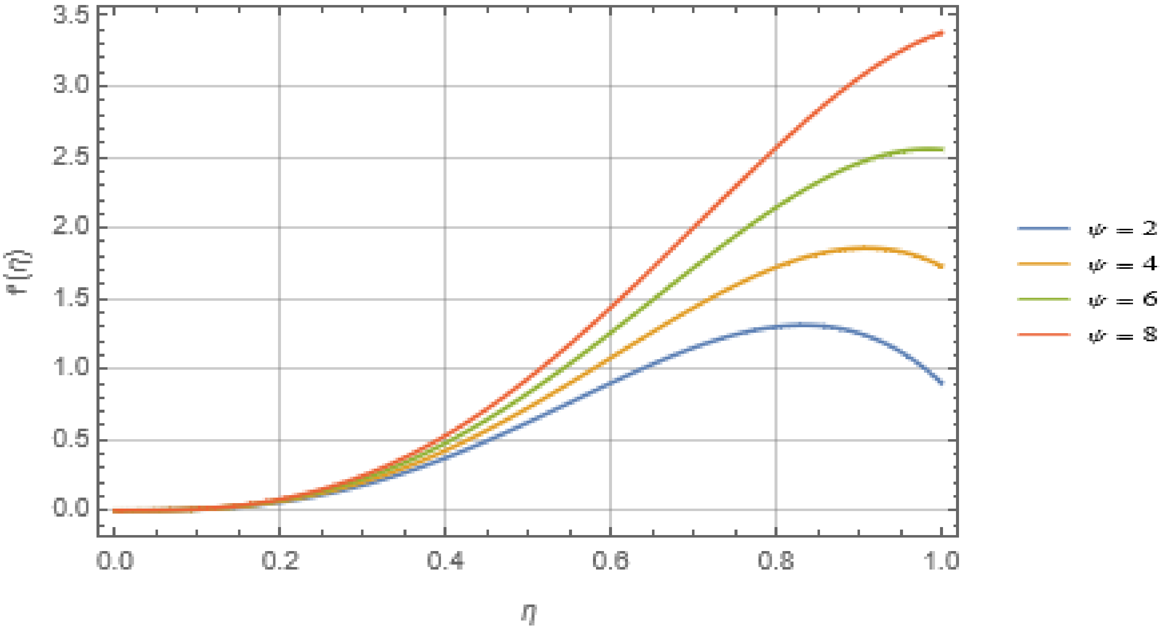

Figure 4. Variation of $f'(\eta)$ with η for different values of porosity parameter

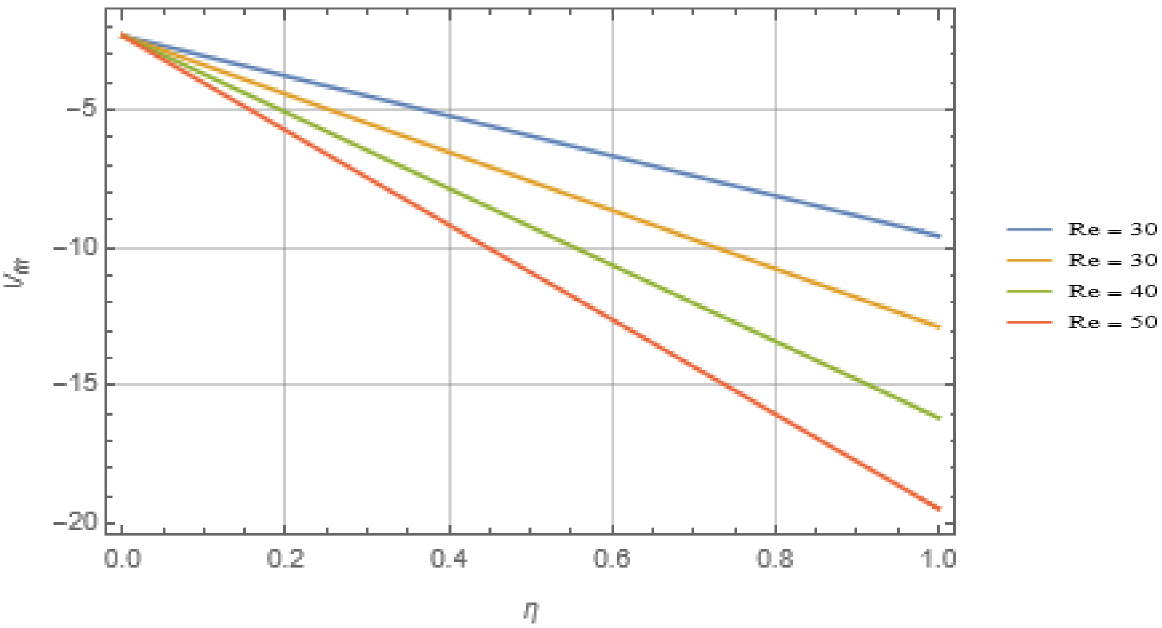

Figure 5. Variation of V_m with η for different values of Reynold number

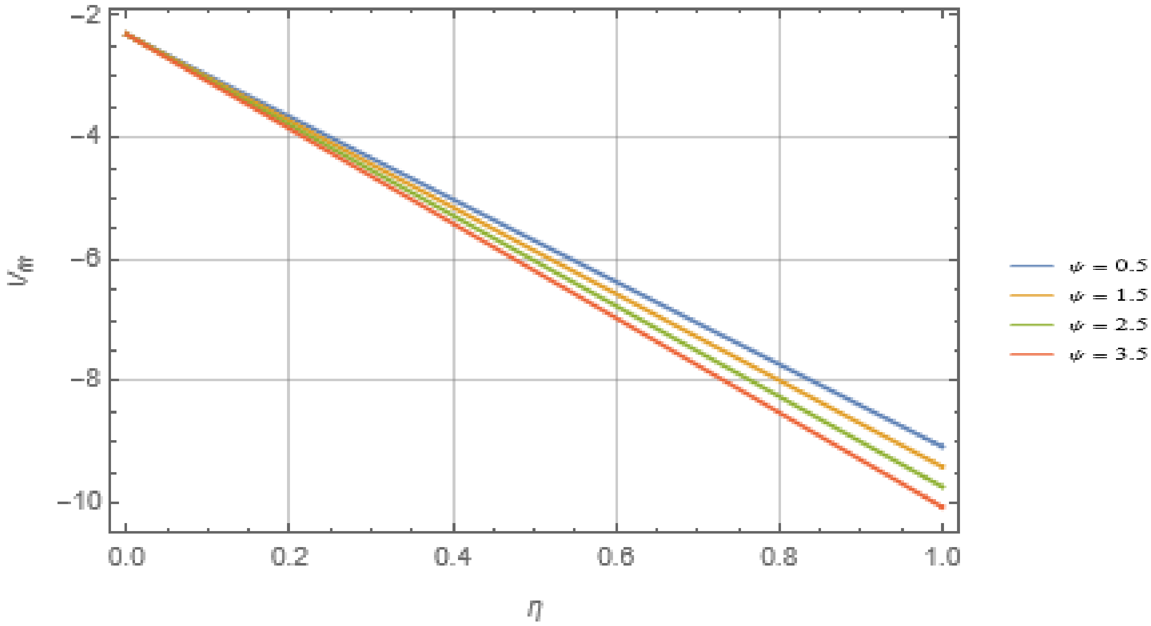

Figure 6. Variation of V_m with η for different values of the parameter, ψ

VIII. CONCLUSION.

In this research article, we analytically investigate the transport of chemical ion through the soil. The governing equations were transformed to a nonlinear ordinary differential equation, where they were solved using Variational Homotopy perturbation method. The findings of thearticle are summarized as follows.

(i) Increase in Reynold number, Re leads to decrease in the temperature profile, whereas the reverse is also true.
(ii) Decrease or increase in porosity parameter leads to a decrease and increase in the temperature profile of the velocity profile.
(iii) The velocity profile increases with an increase in the Reynold and porosity parameter
(iv) The volumetric flow rate for both temperature and velocity profiles decrease for increase in both porosity and Reynold numbers.

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NOMENCLATURE

(x, y)	Cartesian coordinates of a point
(u, v)	Velocity component along $x - and\ y -$directions

U_0	Characteristic velocity
F_0	Applied external force
k	Permeability of the porous medium
h	Half-width of the channel
η	Non-dimensional distance
μ	Coefficient of viscosity
ν	Kinematic viscosity of solute
ρ	Density of solute
Re	Reynold number
K	Permeability parameter