Optimal design of Delta-type parallel robots based on optimal simultaneous parameters related to transmission pressure angle and Jacobian matrix

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ABSTRACT

The problem of optimal design of parallel robots, especially multi-objective optimization, is a cumbersome and difficult problem to solve. First of all, because the scale of the problem is very large, the number of input parameters as well as the number of quality indicators used to evaluate the design is very large. In this article, there are 7 criteria that need to be optimized at the same time. For the purpose of determining the structural dimensions of the robot so that the performance benefits and transmission ratios of the robot are maximized, the author uses the MCDM method to synthesize the size of a Delta robot when gradually increasing the number of design criteria from 3 to 7 different criteria. The paper also illustrates how the method handles the original data from the cloud to ensure that this data aligns with the MCDM problem. The entire process of preparing data for optimal design criteria, including 7 criteria W, ITI, OTI, LTI, K, $\eta_{v\min}, \eta_{p\min}$, is linked to the reverse dynamics problem of the robot and automates the process of solving this problem to accelerate the entire design process. It is this characteristic that creates the significant improvements to the design productivity. Especially, this method allows the inclusion of design indicators of different nature into a single problem. This is an issue that a method such as Atlas is unable to address.

Keywords—*Parallel Robot, ITI, OTI, LTI, Delta, MCDM.*

I. INTRODUCTION

Single-objective and multi-objective optimal design is widely applied in parallel robot design. Multi-objective problems are useful when synthesizing the dimensions of a parallel robotic mechanism. For kinetic synthesis, people often know the principal diagram of the robot in advance but will not have the size of its stitches. Unlike high articulation delivery systems, the parallel robot is a low articulation delivery system. These structures have transmission efficiency (ITI input transmission coefficient, OTI output transmission coefficient, LTI global transmission coefficient) and transmission ratio (force, velocity) between the joint space and the workspace that varies according to the structure size of the robot and with each defined structure, these parameters also change throughout the workspace of the robot [1] [2]. With the characteristics of these indicators depending on the size of the structure and the coordinates of the survey point in the workspace, these indicators cannot be affected through control but can only be optimized at the dynamic synthesis stage of the robot.

Basically, optimal kinetic synthesis studies often revolve around two systems of quality criteria, the system of criteria related to transmission pressure angle and the Jacobian matrix related criteria system. Combining all these indicators into a single problem like our goal here is quite rare. Typically, studies consider a system of criteria among the two systems [1], [2], [3]. The method for surveying a multi-objective problem in that case can be graphical, trend zoning combined with result reasoning [4], [5], [6], [7], [8]. It can be the structure of a large multi-objective problem then solved by SPEA II or GA [9], [10], it can also be broken down into smaller-scale problems solved separately then piled up by Atlas method [4], [11], [12], [13], [14]

At even larger scales of data, such as consideration of customer needs, typical needs and benefits, economics, fabricability, material use, and the smallest size that satisfies the constraints. One can completely set up an input data matrix as in[9], [10], which is solved by a genetic algorithm.

Methodologically, the Atlas method is only suitable for problems where design criteria are related to Jacobian matrices. This is because it has been proven that the homogeneous relationship between the basic plan and the homogeneous plan is identical to the indicators derived from the Jacobian matrix [12]. Thus, this method cannot take into account the indicators belonging to the group related to the pressure transmission angle. The Atlas method itself does not solve asymmetrical structures because they have more than 4 parameters to identify. Its variable surface structure allows only one dependent parameter to be removed and the remaining three parameters are equal to the number of dimensions of the space that can be built graphically.

The GA method can be a more universal method when it only needs to establish the objective function regardless of which system the criterion comes from and how the dimension is. Its limitation lies in the fact that the solution of this method is unique. There is no alternative to that unique solution that sometimes leads to difficulties in pushing the problem further when there are other technological requirements to incorporate into the solution [10].

The Atlas method and the GA method both produce a global optimal solution, but they belong to two completely different perspectives. If GA sees the optimal structure as a large, intact problem, the Atlas method will break down the original problem into separate problems with lower complexity and solve them separately. The final results are piled up to determine the optimal domain. The problem solved by the GA method takes place only in the robot's workspace, while the problem solved by Atlas takes place in two phases. Phase 1 is optimal in the dimensionless domain called the design space, Phase 2 maps the result from the dimensionless domain to the dimensionless domain to the dimensioned workspace.

In the general picture of the multi-purpose design method applied to parallel robots as analyzed above, there may be one more method missing. This method must be able to solve the multi-criteria problem regardless of the origin of that criterion on the basis of the pressure angle system or Jacobian matrix as the GA method has done. It is not limited by the number of input criteria and the number of parameters to be defined at the output as the Atlas method and especially limits the weaknesses of the GA method such as the optimal number of targets and is quite limited at the same time. For example, in , the problem presents only two objectives. This number is quite small for the design needs and while the program is quite complex. This limits its widest applicability to problems with an even larger number of goals.

With data sets where the difference between the indicators is unclear, the application of more than one multi-criterion decision-making method can lead to a change in ranking or not will also be clear in this article.

II. System of design criteria, expressions and meanings

2.1 Indicators related to the transmission pressure angle

In parallel robots, the power from the fixed platform is transmitted to the mobile platform by various branches. Because the stages in the chain are connected by a rheumatoid joint that constantly changes direction when working, the power transmission efficiency in the branch changes according to the posture between them. See figure 1 below.

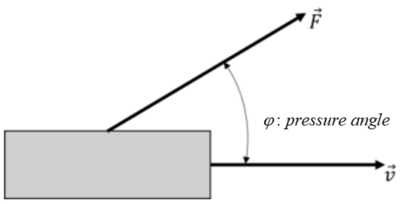

Fig1. Description of infusion pressure angle

Assume that due to a change in posture during work, the force F acting on the stitch does not coincide with its direction of motion, so instead of the entire F involved in driving, only the component involved in driving. is the instantaneous power loss factor that describes the performance of that drive. When in different positions, this loss is different, and the maximum force applied at this end of the branch can only be as large as it remains at the end of the branch or smaller than itself due to the loss along the branch and not greater than the value applied to the end.

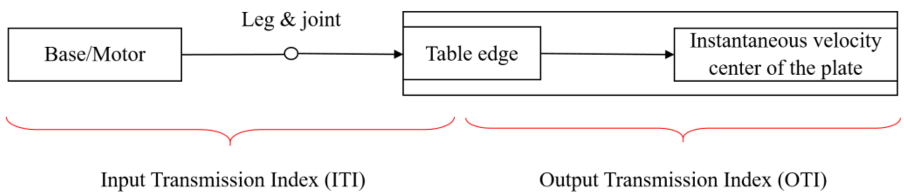

Fig 2. Description of drive stage

With sufficiently driven parallel robots there will be two stages of transmission:

- The first stage of transmission or input from the fixed platform to the edge of the mobile platform (ITI);

- The second stage of transmission from the edge of the table to the instantaneous velocity center of the table (OTI);

Since the performance of these two stages can be different, the concept of instantaneous global transmission coefficient (LTI) is evaluated with the smallest of those two values.

To calculate these specific indicators, it is necessary to consider the basis from screw algebra.

Let the pin-active screw i denotes $\$_{Ii}$ that it is the cause of the movement of this branch, resulting from the active action of the motor on that branch.

Under the impact of $\$_{Ii}$ there will be $\$_{wi}$ a force wrench at the next stage with the active stage. The $\$_{wi}$ can be considered as a direct consequence of the drive of the screw $\$_{Ii}$. Between the causal pair $(\$_{Ii}, \$_{wi})$ exists the driving efficiency given by (1) characterizing the power loss in the first stage arm of the transmission:

$$ITI = \frac{|\$_{Ii} \circ \$_{wi}|}{|\$_{Ii} \circ \$_{wi}|_{\max}} \quad (i = 1,2,3) \tag{1}$$

Suppose, in turn, that it is the cause of the kinetic movement of the symbolic mobile pedestal in the next stage, between the causal pair – this result again exists a performance of the second stage given by (2):

$$OTI = \frac{|\$_{wi} \circ \$_{Oi}|}{|\$_{wi} \circ \$_{Oi}|_{\max}} \quad (i = 1,2,3) \tag{2}$$

The values of OTI and ITI are both in the range of01 and therefore the power gradually decreases with transmission. Overall loss of the entire drive line assessed by (3)

$$LTI_i = \min (OTI_i, ITI_i) \tag{3}$$

The coefficients in (1,2,3) are called the performance of the parallel robotic mechanism. It is generally desirable to design a parallel robot with high transmission efficiency throughout its workspace. This will continue to be reflected in the optimal problem in section 4.

2.2 Criteria related to Jacobian matrix

If the indicators related to the transmission pressure angle in section 2.1 show that the power loss includes the velocity loss and the force in the transmission branch occurring in the workspace of the robot i.e. the space of the Oxyz reference system, there is another relationship that needs to be studied in the optimal problem that is the transmission ratio.

Let x' be the final impact stitch velocity in the Oxyzworkspace.

Let q' be the articulation velocity in the articulation space of the robot.

Referring to a driving moment in a joint space.

Let F be the attenuating force on the final impact in the Oxyzworkspace.

The relationships mentioned here are the ratio of their transition from one space to another.

For the first pair of relations (x', q'). Suppose in the Oxyz workspace there is a need for displacements with unit velocities in any direction described as a radius sphere 1 as in (4):

$$x'^T . x' = 1 \tag{4}$$

substitue $x' = J^{-1}.q'$ into (4) to get (5):

$$x'^T x' = q'^T (J^{-1})^T J^{-1} q' \quad (5)$$

Due to $(J^{-1})^T.(J^{-1})$ is semi-positive, its eigenvalues $\lambda_1, \lambda_2, \lambda_3$ coincide with the xyz directions of the reference frame and are equal. When mapping the sphere (4) back to the joint space there is the following situation:

- $\lambda_1 = \lambda_2 = \lambda_3$ denotes a sphere, i.e. the velocity sphere of diameter $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ in the joint space, mapped to a unit sphere in the workspace. This is the ideal case, how many points in the workspace appear depends on the structure of the robot itself.
- $\lambda_1 = \lambda_2 \neq \lambda_3$ denotes a circular ellipsoid (the baseline is a circle), having a semi-major axis of $\lambda = \lambda_1 = \lambda_2$ and a semi-major axis of λ_3;
- $\lambda_1 \neq \lambda_2 \neq \lambda_3$ denotes a flattened ellipsoid having three different axes.

In the case of an ellipsoid with a large radius difference between the semi-axes, the ellipsoid can approximate a cylinder with a radius. Then, if you want to create a velocity of 1 unit in the xyz workspace, the impactor in the qi joint space needs to create an extremely large velocity, which is not feasible, it is outside the capacity of the motor. Denote $\lambda \min$ In the case of an ellipsoid with a large radius difference between the semi-axes, the ellipsoid can approximate a cylinder with a radius. Then, if you want to create a velocity of 1 unit in the xyz workspace, the impactor in the qi joint space needs to create an extremely large velocity, which is not feasible, it is outside the capacity of the motor $(J^{-1})^T.(J^{-1})$ it denotes the minimum velocity of the transmission, the maximum desired minimum velocity transmission factor to maximize the labor productivity of the robot. That is the first indicator related to the Jacobian matrix.

For the second pair of relations $J^T.J$ the (τ, F) product represents a force ellipsoid. Suppose there is a unit impactor in the form of a sphere in space that matches radius 1. When mapping to the workspace in case the three eigenvalues of the ellipsoid are $J^T.J$ equal, we get a sphere. This is the most ideal case. More commonly we will get an ellipsoid with its own values $\lambda_1 \neq \lambda_2 \neq \lambda_3$.

Thus, in the direction of deformation, the structure is the smallest, or in other words, the firmness of the structure is the largest in this semi-axial direction. In contrast, in the direction of the structure with the lowest bearing capacity in this direction. Therefore, it is necessary to strengthen small deformation points throughout the workspace when designing optimally rather than accepting large deformation points.

The third indicator associated with Jacobian matrices is the number of conditions K.

Dexterity of robots is an important indicator when designing robots in parallel. It is also known as the movement regulation index of the robot, which shows the ability of the robot to drive isotopically in a specific position in the workspace. The first is the number of conditions of the Jacobian matrix defined through the robot's Jacobian matrix type two standard:

$$K_2 = \|J\|_2 . \|J^{-1}\|_2 \quad (6)$$

Thus, it is necessary to first identify J from the configuration of the robot, from J ➔ J^{-1};

Assuming:

$$J = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad \text{và} \quad J^{-1} = \begin{vmatrix} e & f \\ g & h \end{vmatrix} \tag{7}$$

Calculate norm of J and J^{-1} horizontally (second norm or infinitive norm):

$$\begin{aligned} \|J\| &= \max\{(|a|+|b|),(|c|+|d|)\} \\ \|J^{-1}\| &= \max\{(|e|+|f|),(|g|+|h|)\} \end{aligned} \tag{8}$$

The greater the dexterity index of the robot, the more dexterous the robot is, so when designing optimally, it is necessary to maximize this index. Unlike a performance index group, the force or velocity transformation ratio between two unconfined spaces is less than 1, which can have any positive value.

III. MCDM METHOD

3.1 Sequence and optimal design methodology for maximum objectives

When calculating the initial design criteria, we mesh the entire work area of the robot, the evenly spaced nodes are located at the top of the cubes over the entire work area. These points are included in the reverse kinematics problem solving to determine which points are in the workspace and which are outside the workspace. Because the quality indicators presented in section 2 are related to the configuration, the survey results in this step are necessary to calculate the values of ITI, OTI, LTI, K, at each node in the work area and count the number of W nodes in the work area. However, this data cannot be included in the MCDM problem as the algorithm of choice for handling the maximum target optimization problem in this study. After all the ITI, OTI, LTI, K data $\eta_{v\min}, \eta_{p\min}$ has been achieved, their zoning statistics across the domain, for example, the ITI 0.7 threshold represents the number of nodes whose ITI reaches the value of 0.7, at which point the cloud data translates into a value that represents the desired quality of the robot in that aspect. This index is suitable for inclusion in the optimal problem and the same treatment with all remaining indicators OTI, LTI, K, $\eta_{v\min}, \eta_{p\min}$ is necessary. Particularly for W if the embodiments to be compared to each other all use the same single point grid structure, it is only necessary to count the number of nodes in the work area to compare which embodiment has an advantage instead of calculating its volume.

3.2 Sequence and optimal design methodology for maximum objectives

3.2.1 Optimal design sequence for maximum objectives

The problem of optimal design of parallel robots using 7 quality criteria is described as follows. From the identified kinetic structure but the suture does not have the optimal size. Let l_i with i = 1,2,.. is the size to be synthesized according to the 7 criteria presented in section 2 above. Referring to the embodiments$li^{(a)}$,$li^{(b)}$,...as structural embodiments on the basis of a known structure with the view that the branches of these embodiments have the same total length. The best of the ITI, OTI, LTI, K, $\eta_{v\min}, \eta_{p\min}$ and W options should be identified.

Diagram in Fig 3 shows the sequence of the problem execution:

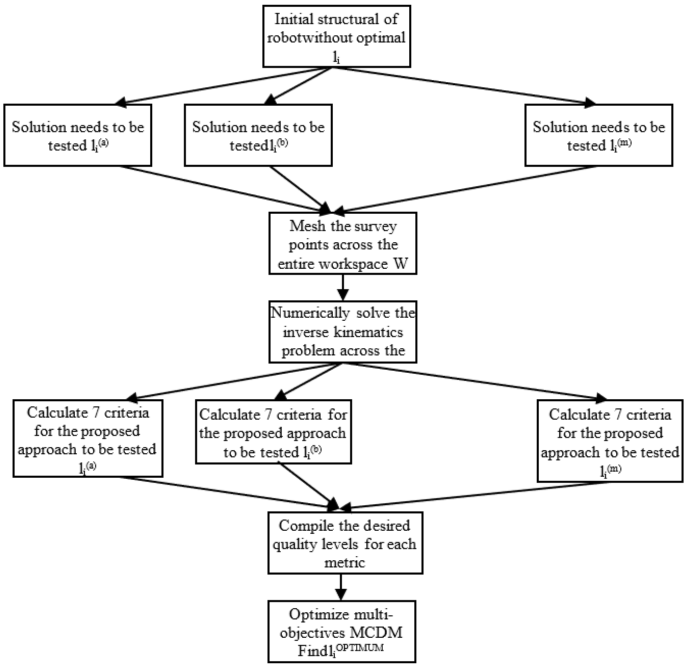

Fig 3. Multi-objective optimization sequence

3.2.2 EDAS method

EDAS Distance from Average Solution (EDAS) method is an MCDM method that was first introduced in 2015. The steps to multi-criteria decision making according to the EDAS method are as follows[15], [16]:

Step 1: Building a decision matrix based on the following formula:

$$X = \left[x_{ij}\right]_{mxn} = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ x_{21} & \dots & x_{2n} \\ & \dots & \\ x_{mn} & \dots & x_{mn} \end{bmatrix} \quad (9)$$

Where: m is the number of options, n is the number of criteria, $x_{ij}$is the value of criterion j adoptioni.

Step 2: Determining an average (AVG) of the options.

$$AVG = \frac{\sum_{i=1}^{m} x_i}{m} \quad (10)$$

Step 3: Defining positive distance (PD)and negative distances (ND) from an average.

$$PD_{ij} = \frac{\max\left[0,\left(x_{ij} - AVG_j\right)\right]}{AVG_j} \; \textit{if j is an indicator whose value is as high as possible} \quad (11)$$

$$PD_{ij} = \frac{\max\left[0,\left(AVG_j - x_{ij}\right)\right]}{AVG_j} \text{ if } j \text{ is an indicator whose value is as low as possible} \quad (12)$$

$$ND_{ij} = \frac{\max\left[0,\left(AVG_j - x_{ij}\right)\right]}{AVG_j} \text{ if } j \text{ is an indicator whose value is as high as possible} \quad (13)$$

$$ND_{ij} = \frac{\max\left[0,\left(x_{ij} - AVG_j\right)\right]}{AVG_j} \text{ if } j \text{ is an indicator whose value is as low as possible} \quad (14)$$

Step 4: Calculating the sum of positive distances (SoP) and the sum of negative distances (SoN).

$$SoP_i = \sum_{i=1}^{m} w_j.PD_{ij} \quad (15)$$

$$SoN_i = \sum_{i=1}^{m} w_j.ND_{ij} \quad (16)$$

where w_j is the weight of the criterion *j*.

Step 5: Normalizing the SoP and SoN values according to the formula.

$$SSoP_i = \frac{SoP_i}{\max(SoP_i)} \quad (17)$$

$$SSoN_i = 1 - \frac{SoN_i}{\max(SoN_i)} \quad (18)$$

Step 6: Calculating appraisement score (APS) of the options based on the formula.

$$APS_i = \frac{1}{2}\left(SSoP_i + SSoN_i\right) \quad (19)$$

Step 7: Ranking the options according to the rule that the option with the highest $APS_i$is considered the best.

3.2.3 Determine the weights using the Entropy method

In this study, the weights of the criteria were determined by the Entropy weight method [18]. The steps to calculate weights are given (24) đến (26):

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} \quad (24)$$

$$e_j = -\sum_{i=1}^{m}\left[p_{ij} \text{ x} \ln\left(p_{ij}\right)\right] - \left(1 - \sum_{i=1}^{m} p_{ij}\right) \text{ x} \ln\left(1 - \sum_{i=1}^{m} p_{ij}\right) \quad (25)$$

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}\left(1-e_j\right)} \tag{26}$$

IV. CASE STUDY

4.1 Partition of indicators

This section shows the process of optimally designing a delta-type parallel robot according to the method mentioned above.

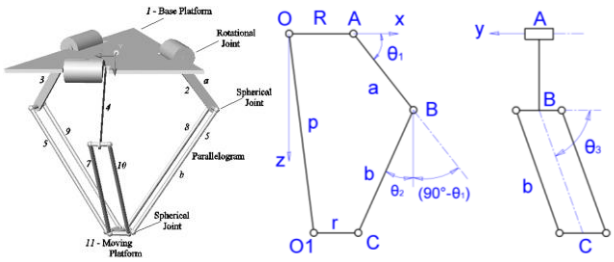

Fig.4. Delta robot and its design parameters

The reverse kinematic problem of the robot is solved by the GRG method [19] and fully automated by VBA as shown in Figure 5.

The notion that R and r cross each other in general, the direction between the two reference frames O and O1 is different once rotated by the R_{RPY} matrix. The vector ring equation for the pentagon has the following form:

$$\vec{p} + R_{RPY}.\vec{r} - \vec{R} = \vec{a} + \vec{b} \tag{27}$$

The graphic below is to calculate the detailed expansion coordinates for the right side:

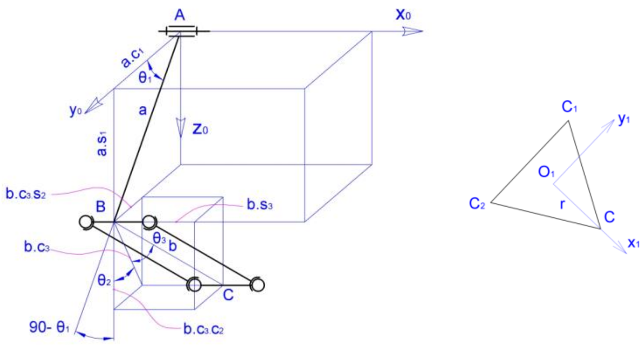

Fig 5. Support for calculating coordinate of C_i point

The virtual vector ring equation written for the robot's branches has the following form:

Leg 1:

$$\begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix} + \begin{vmatrix} c\alpha.c\beta & c\alpha.s\beta.s\gamma - s\alpha.c\gamma & c\alpha.s\beta.c\gamma + s\alpha.s\gamma \\ s\alpha.c\beta & s\alpha.s\beta.s\gamma + c\alpha.c\gamma & s\alpha.s\beta.c\gamma - c\alpha.s\gamma \\ -s\beta & c\beta.s\gamma & c\beta.c\gamma \end{vmatrix} . \begin{vmatrix} r \\ 0 \\ 0 \end{vmatrix} - \begin{vmatrix} R \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 + b.s_{31} \\ a.c_{11} - b.c_{31}s_{21} \\ a.s_{11} + b.c_{31}.c_{21} \end{vmatrix} \tag{28}$$

Leg 2:

$$\begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix} + \begin{vmatrix} c\alpha.c\beta & c\alpha.s\beta.s\gamma - s\alpha.c\gamma & c\alpha.s\beta.c\gamma + s\alpha.s\gamma \\ s\alpha.c\beta & s\alpha.s\beta.s\gamma + c\alpha.c\gamma & s\alpha.s\beta.c\gamma - c\alpha.s\gamma \\ -s\beta & c\beta.s\gamma & c\beta.c\gamma \end{vmatrix} . \begin{vmatrix} \frac{-r}{2} \\ \frac{r\sqrt{3}}{2} \\ 0 \end{vmatrix} - \begin{vmatrix} \frac{-R}{2} \\ \frac{R\sqrt{3}}{2} \\ 0 \end{vmatrix} = \begin{vmatrix} 0 + b.s_{32} \\ a.c_{12} - b.c_{32}s_{22} \\ a.s_{12} + b.c_{32}.c_{22} \end{vmatrix} \tag{29}$$

Leg 3:

$$\begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix} + \begin{vmatrix} c\alpha.c\beta & c\alpha.s\beta.s\gamma - s\alpha.c\gamma & c\alpha.s\beta.c\gamma + s\alpha.s\gamma \\ s\alpha.c\beta & s\alpha.s\beta.s\gamma + c\alpha.c\gamma & s\alpha.s\beta.c\gamma - c\alpha.s\gamma \\ -s\beta & c\beta.s\gamma & c\beta.c\gamma \end{vmatrix} . \begin{vmatrix} \frac{-r}{2} \\ \frac{-r\sqrt{3}}{2} \\ 0 \end{vmatrix} - \begin{vmatrix} \frac{-R}{2} \\ \frac{-R\sqrt{3}}{2} \\ 0 \end{vmatrix} = \begin{vmatrix} 0 + b.s_{33} \\ a.c_{13} - b.c_{33}s_{23} \\ a.s_{13} + b.c_{33}.c_{23} \end{vmatrix} \tag{30}$$

To construct a Jacobian matrix of a delta robot, from a virtual vector ring equation of the form:

$$\vec{p} + R_{RPY}.\vec{r} - \vec{R} = \vec{a} + \vec{b} \tag{31}$$

Derivative both sides over time to obtain:

$$(\overrightarrow{p'} + \overrightarrow{r'}) = \overrightarrow{a_i'} + \overrightarrow{b_i'} \tag{32}$$

In which, R is considered as a constant because it describes the fixed pedestal structure, when the derivative will disappear. In the expression above the points on the mobile platform will have the same velocity:

$$\overrightarrow{p'} = \vec{v} = \overrightarrow{a_i'} + \overrightarrow{b_i'} \tag{33}$$

Or expressed in terms of angular velocity will take the form of:

$$\vec{v} = \overrightarrow{\omega_{ai}} \times \overrightarrow{a_i} + \overrightarrow{\omega_{bi}} \times \overrightarrow{b_i} \tag{34}$$

multiply both sides of the aforementioned equation with the unit vector $\hat{b}_i$:

$$\hat{b}_i.\vec{v} = \hat{b}_i.[\overrightarrow{\omega_{ai}} \times \overrightarrow{a_i} + \overrightarrow{\omega_{bi}} \times \overrightarrow{b_i}] \tag{35}$$

Since the perpendicular scalar product component is zero, the other expression takes the form:

$$\hat{b}_i.\vec{v} = \hat{b}_i.\overrightarrow{\omega_{ai}} \times \overrightarrow{a_i} \tag{36}$$

Expand both sides of the above equation to obtain:

$$\begin{aligned}&\hat{b}_i.\vec{v} = [\sin\theta_{3i}\cos(\theta_{2i}+\theta_{1i})][v_x\cos\phi_i - v_y\sin\phi_i]\\&+\cos\theta_{3i}[v_x\sin\phi_i + v_y\cos\phi_i]\\&+[\sin\theta_{3i}\sin(\theta_{2i}+\theta_{1i})]v_z = J_{ix}v_x + J_{iy}v_y + J_{iz}v_z\end{aligned} \tag{37}$$

In which:

$$\begin{aligned}J_{ix} &= \sin\theta_{3i}\cos(\theta_{2i}+\theta_{1i})\cos\phi_i + \cos\theta_{3i}\sin\phi_i\\J_{iy} &= -\sin\theta_{3i}\cos(\theta_{2i}+\theta_{1i})\sin\phi_i + \cos\theta_{3i}\cos\phi_i\\J_{iz} &= \sin\theta_{3i}\sin(\theta_{2i}+\theta_{1i})\end{aligned} \tag{38}$$

In Equations (32) and (33) because the x-axis of the base frame of reference passes through the A1 point of the fixed pedestal, the angle $\phi_i = 0$ [1].

On the right-hand side of equation (27), the motion of joint a lies within the Oxizi plane. Therefore, it has only the velocity component in this plane. This is the angular velocity around the oy axis, hence:

$$\overrightarrow{\omega_{ai}} = \begin{bmatrix}0\\-\theta_{1i}^{'}\\0\end{bmatrix} \tag{39}$$

Or:

$$\overrightarrow{\omega_{ai}} \times \vec{a_i} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\0 & -\theta_{1i}^{'} & 0\\a_{1i} & a_{2i} & a_{3i}\end{vmatrix} = -a_{3i}\theta_{1i}^{'}\hat{i} + a_{1i}\theta_{1i}^{'}\hat{k} \tag{40}$$

The left side of Equation (27) can be simply written as follows:

$$\hat{b}_i(\overrightarrow{\omega_{ai}} \times \vec{a_i}) = -a\sin\theta_{2i}\sin\theta_{3i}\theta_{1i}^{'} \tag{41}$$

Combine Equations (35) and (36) to have:

$$\begin{aligned}J_{1x}v_x + J_{1y}v_y + J_{1z}v_z &= -a\sin\theta_{21}\sin\theta_{31}\theta_{11}^{'}\\J_{2x}v_x + J_{2y}v_y + J_{2z}v_z &= -a\sin\theta_{22}\sin\theta_{32}\theta_{12}^{'}\\J_{3x}v_x + J_{3y}v_y + J_{3z}v_z &= -a\sin\theta_{23}\sin\theta_{33}\theta_{13}^{'}\end{aligned} \tag{42}$$

Rewrite in general form:

$$J_p.\vec{v} = J_\theta\overrightarrow{\theta'} \tag{43}$$

where:

$$J_p = \begin{vmatrix} J_{1x} & J_{1y} & J_{1z} \\ J_{2x} & J_{2y} & J_{2z} \\ J_{3x} & J_{3y} & J_{3z} \end{vmatrix} \quad (44)$$

and:

$$J_\theta = a \begin{vmatrix} \sin\theta_{21}\sin\theta_{31} & 0 & 0 \\ 0 & \sin\theta_{22}\sin\theta_{32} & 0 \\ 0 & 0 & \sin\theta_{23}\sin\theta_{33} \end{vmatrix} \quad (45)$$

Then the Jacobian of the Robot is calculated:

$$J = J_\theta^{-1}.J_p \quad (46)$$

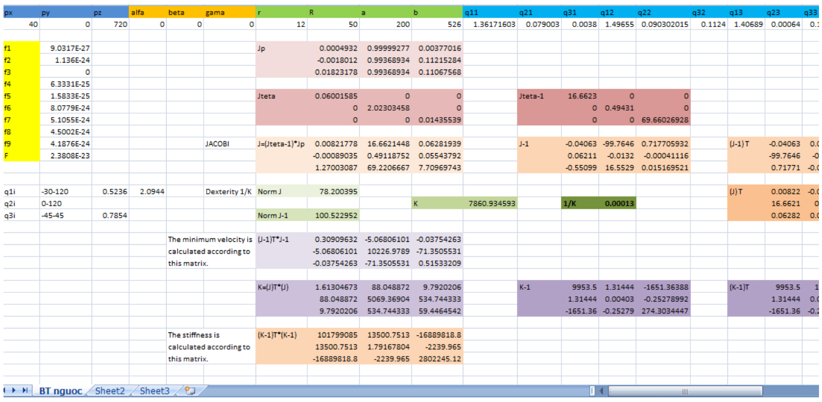

Fig 6. Program interface for calculating delta robot design criteria

In Figure 6, the results of the reverse kinematics problem, the generalized coordinates and the robot's li parameters, matrices $(J^{-1})^T.(J^{-1})$, $K_2 = \|J\|_2.\|J^{-1}\|_2$ and $J^T.J$ prepare inputs for the calculation criteria $\eta_{v\min}, \frac{1}{K}, \eta_{p\min}$. In which, the eigenvalues of $(J^{-1})^T.(J^{-1})$ and $J^T.J$ only take these matrices and move to the calculation in another program as seen in Figure 7.

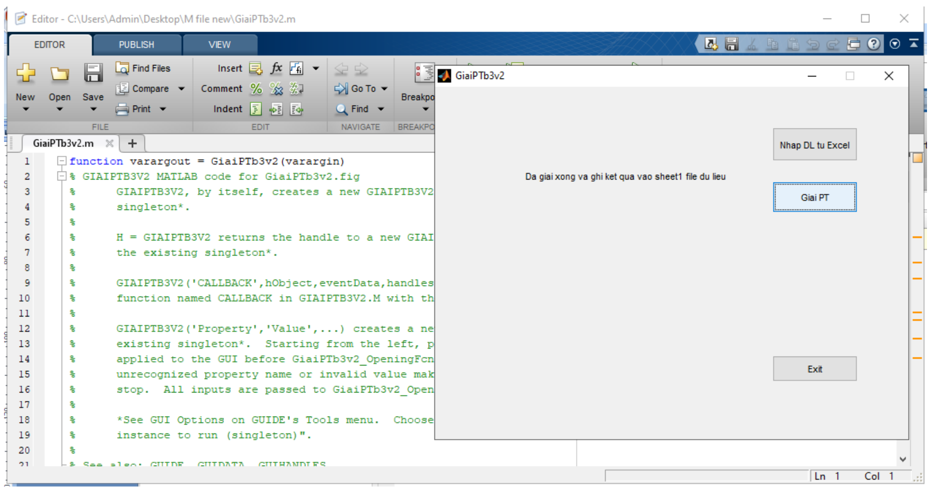

Fig 7. Program to calculate eigenvalues of matrix $(J^{-1})^T.(J^{-1})$ and $J^T.J$ using Matlab

Assume the total leg length of the robot a + b = 726, static pedestal size R = 0.247a, dynamic pedestal r = 0.25R. An option with (a,b,R,r)^{OPT} need to be selected.

Identify 6 options to be tested as shown in Table 1.

Table 1: Optimal survey options

Geometric ratio needs to be optimized a/b/R/r	W	ITI ≥ 0.4	OTI ≥ 0.1	LTI ≥ 0.1	1/K ≥ 0.007	$\eta_{v\min} \geq 25$	$\eta_{p\min} \leq 14000$
200/526/50/12	1799	366	1017	990	716	391	1181
250/476/60/15	2398	1068	2123	2046	1317	1039	2081
300/426/74/18.5	2957	2067	2887	2843	2060	1981	1965
350/376/86/22	3442	1746	3210	3140	1700	1698	2843
400/326/100/25	3055	2777	3038	3011	2534	2631	768
425/301/105/26	2071	2467	2687	2661	1	1	2071

4.2 Application of the EDAS method

Use the EDAS method to rank projects

Step 1: Develop the matrix X according to the structural survey data table (Table 1)

Step 2: Calculate the weight wj according to the formula from (24) to (26), the results are shown in Table 2.

Tab.2. Table of criteria w_j-weighted values

	W	ITI ≥ 0.4	OTI ≥ 0.1	LTI ≥ 0.1	1/K ≥ 0.007	$\eta_{v\min} \geq 25$	$\eta_{p\min} \leq 14000$
w_j	1799	366	1017	990	716	391	1181

Step 3: Apply the formula (10) to determine the average value (AVG) shown in Table 3.

Tab.3. Table of AVG average values of the criteria

	W	ITI ≥ 0.4	OTI ≥ 0.1	LTI ≥ 0.1	1/K ≥ 0.007	$\eta_{v\min} \geq 25$	$\eta_{p\min} \leq 14000$
AVG	2620.3	1748.5	2493.6	2448.5	1388	1290.2	1818.2

Step 4: PDij and NDij values are calculated according to the formula from (11) to (14) provided that the indicators reach the maximum possible value. The calculation results are summarized in Table 4.

Tab.4. The values of PDij and NDij of the criteria

No.	PDij							NDij						
	W	ITI ≥ 0.4	OTI ≥ 0.1	LTI ≥ 0.1	1/K ≥ 0.007	$\eta_{vmin} \geq 25$	$\eta_{pmin} \leq 14000$	W	ITI ≥ 0.4	OTI ≥ 0.1	LTI ≥ 0.1	1/K ≥ 0.007	$\eta_{vmin} \geq 25$	$\eta_{pmin} \leq 14000$
A1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.8	0.6	0.6	0.5	0.7	0.4
A2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.4	0.1	0.2	0.1	0.2	0.0
A3	0.1	0.2	0.2	0.2	0.5	0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A4	0.3	0.0	0.3	0.3	0.2	0.3	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A5	0.2	0.6	0.2	0.2	0.8	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6
A6	0.0	0.4	0.1	0.1	0.0	0.0	0.1	0.2	0.0	0.0	0.0	1.0	1.0	0.0

Step 5: The values SoP_i, SoN_i,$SSoP_i$, $SSoN_i$,APS_i in table 5 are calculates using formula (15) to (19). Ranking results of the embodiments are also presented in this table.

Tab.5. Parameters and ranking results according to the EDAS method

No.	SoPi	SoNi	SSoPi	SSoNi	APSi	Rank
A1	0	0.54619	0	0	0	6
A2	0.02064	0.14755	0.04713	0.72984	0.388485	4
A3	0.24706	0	0.56404	1	0.78202	3
A4	0.28397	0.0002	0.64832	0.99963	0.82397	2
A5	0.43801	0.0825	1	0.84899	0.92449	1
A6	0.10203	0.31523	0.23295	0.42277	0.32786	5

The EDAS method indicates that option A5 with structural indicators a=350, b=376, R=86, r=22 is the best choice for the working area with outstanding OTI, LTI and hardness indicators. While embodiment A4 with structural indices a=400, b=326, R=100, r=25 is the 2nd suitable option

V. Conclusion

It is very important to have a method that can handle multiple optimal design criteria of a parallel robot at the same time. Parallel robots have a narrow working area, the design process needs to make the most of the beneficial indicators and limit the unfavorable characteristics for the robot. Since the control technique itself can only affect the accuracy and productivity but not the transmission efficiency and the force/ velocity conversion ratio, the optimal problem at the same time all 6 indicators to determine the performance and transmission ratio are very important and need to be solved right in the structural design phase of the parallel robot as shown here.

Although the MCDM method does not give global optimal results like Atlas or GA, it allows to consider many optimal indicators at the same time while GA rarely considers more than 3 indicators. MCDM also does not limit the number of parameters to synthesize while Atlas is limited to no more than 4 parameters.

With such prospects, MCDM can be used for new design or selection of parallel robots when applied for specific purposes. MCDM can also be used in combination with Atlas in a two-stage problem in which Atlas has the function of optimizing the transmission ratio indicators while MCDM considers that around the global optimal plan created by Atlas, which plan will have the best performance criteria.

When applying more than one MCDM method on the same data set, the rank reversal occurred in the first group where the difference between the indicators was not large enough while the rank of the remaining options was kept the same.

ACKNOWLEDGMENT

The authors gratefully acknowledge Thai Nguyen University of Technology, Vietnam, for supporting this work.

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